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### **ABSTRACT**

This paper studies the limitations of monetary policy transmission within a credit channel framework. We show that, under certain circumstances, the credit channel transmission mechanism fails in that liquidity injections by the central bank into the banking sector are hoarded and not lent out. We use the term ‘credit traps’ to describe such situations and show how they can arise due to the interplay between financing frictions, liquidity, and collateral values. Our analysis offers a characterization of the problems created by credit traps as well as potential solutions and policy implications. Among these, the analysis shows how quantitative easing and fiscal policy acting in conjunction with monetary policy may be useful in increasing bank lending. Further, the model shows how small contractions in monetary policy or in loan supply can lead to collapses in lending, aggregate investment, and collateral prices.

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## Introduction

The literature on the credit channel of monetary policy analyzes how changes in the money supply affect real economic activity through their impact on financial frictions and the availability of credit. However, little is known on when and why such a policy will fail to induce bank lending. Our paper fills this gap. Using a general equilibrium model with endogenous collateral values, we show that banks may rationally choose to hoard liquidity during monetary expansions rather than lend it out. Despite the best efforts of the central bank to stimulate lending, liquidity remains trapped in banks. In equilibrium, investment levels do not rise, collateral values remain depressed, and liquidity in the corporate sector remains low. We use the term ‘credit traps’ to describe these scenarios, and show how they can arise due to an adverse interplay between liquidity and the value of collateral.

Our model has two building blocks. The first is the well known notion that collateral eases financial frictions and increases debt capacity (see e.g. Hart and Moore 1994, 1998). The second building block of the model is that the value of firms’ collateral is determined, in part, by the liquidity constraints of industry peers. As in Shleifer and Vishny (1992) we assume that banks cannot operate assets on their own to generate cash flow and so must sell seized collateral to other industry participants. Liquidity constraints in these peer firms, therefore, affect collateral values through their impact on the amount which potential purchasers can pay for assets. In particular, when industry financial conditions are poor, the liquidation value of collateral – which is the relevant value to the bank – might be lower than the intrinsic value of the assets.

Based on these two building blocks, the mechanism of our model hinges on a feedback loop between collateral values, lending, and liquidity in the corporate sector. According to this, increases in collateral values allow greater lending due to the attendant reductions in financial frictions; Greater lending, in turn, increases liquidity in the corporate sector; Finally, increases in corporate liquidity serve to increase collateral values, as these are determined in part by the ability of industry peers to purchase firm assets (Shleifer and Vishny, 1992). Monetary policy affects real outcomes through its impact on this feedback loop between collateral values, lending, and corporate liquidity. By injecting liquidity into the banking sector, monetary policy shifts banks’ lending calculus as they know that increased aggregate lending will influence collateral values.

Our model identifies three mutually exclusive types of potential equilibria of monetary transmission. In the first type of equilibrium, which we call the ‘conventional equilibrium’, shifts in

monetary policy successfully influence aggregate lending activity. This rational expectations equilibrium can be described by the following series of interlocking forces. When the central bank eases monetary policy, the supply of loanable funds increases. Similar to a standard monetary lending channel effect (see e.g. Bernanke and Blinder (1988)), banks will tend to lend out more funds which will increase liquidity in the corporate sector. As liquidity in the corporate sector increases, liquidation value of assets will increase due to a Shleifer and Vishny (1992) effect: firms become less liquidity constrained, and can hence bid more aggressively when acquiring assets of liquidated firms. As in a standard ‘balance sheet channel’ effect (e.g. Bernanke and Gertler (1989, 1990, 1995)), the endogenous increase in liquidation values improves firms’ collateral positions, and thus enables them to borrow the additional liquidity which was injected to the commercial banks by the central bank.

The lending and balance sheet channels of monetary policy are therefore linked in a rational expectations equilibrium through endogenous collateral values: increased bank lending leads to greater liquidity in the corporate sector and thus higher collateral prices. In turn, higher anticipated collateral prices reduce financial frictions and enable banks to utilize the central-bank injection of liquidity to increase lending. In this conventional equilibrium, an easing of monetary policy thus translates into three effects: an increase in lending, an increase in collateral values, and a change in the interest rate associated with bank lending.

The second type of equilibrium in our model is the ‘credit trap’ equilibrium. In this equilibrium, any easing of monetary policy beyond a certain point is completely ineffective in increasing lending – banks simply hold on to the additional reserves created by the central bank. In the credit trap equilibrium aggregate lending is constrained by low collateral values. To increase collateral values the central bank would need to induce banks to inject additional liquidity into the corporate sector so as to increase firms’ ability to purchase the assets of other industry participants. However, the marginal increase in collateral values implied by additional lending, and the associated increase in debt capacity, are not sufficiently large to actually induce banks to lend. Regardless of the amount of liquidity added by the central bank, credit therefore remains stuck in banks and collateral values do not increase beyond the low level implied by the lack of corporate liquidity.

The third equilibrium type in our model is the ‘jump start’ equilibrium. In this equilibrium monetary policy can be effective, but only when the central bank acts sufficiently forcefully in injecting reserves to the banking sector. When increasing reserves by only a moderate amount,

credit remains trapped in the banking sector as in a credit trap equilibrium. Banks rationally understand that when they can employ only a moderate amount of reserves to lend to firms, the implied collateral values are too small to justify any actual lending. Banks, therefore, retain the additional liquidity provided by the central bank as reserves, lending remains low, and in equilibrium the interest rate on loans will remain constant at its lower bound. However, when the central bank eases monetary policy sufficiently, a high lending and high collateral value rational expectations equilibrium arises: lending is high because collateral values are high enough to support it, while collateral values are high because lending increases liquidity in the corporate sector.

The jump start equilibrium, therefore, provides theoretical support motivated by the credit channel framework for a policy of quantitative easing, showing how, under certain circumstances, such easing can be effective in increasing lending.<sup>1</sup> The jump start equilibrium also explains how small contractions in the stance of monetary policy can lead to large crashes in both asset values and lending. According to this, small reductions in lending reduce liquidity in the corporate sector which, in turn, decreases collateral values. Firm balance sheets are therefore weakened, reducing lending still further. Small reductions in aggregate lending induced by monetary policy are thus amplified, thereby bringing about large contractions in equilibrium lending and collateral values. This effect is very much consistent with accounts of the Japanese experience during the 1980s such as Bernanke and Gertler (1995) who argue that “the crash of Japanese land and equity values in the latter 1980s was the result (at least in part) of monetary tightening; ... [T]his collapse in asset values reduced the creditworthiness of many Japanese corporations and banks, contributing to the ensuing recession.”

We show that the nature of the equilibrium that arises – be it a credit trap or jump start equilibrium – crucially depends on the relation between liquidity and the value of collateral. Credit traps are associated with a concave relation between aggregate liquidity and collateral values while convexities in this relation lead to a jump-start equilibrium. By providing micro foundations for the market for assets, we show that lower asset redeployability, or alternatively, search costs in finding a suitable buyer for assets, make quantitative easing equilibria more likely as compared to credit traps. However, expectations of large future asset sales – say due to an economic downturn

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<sup>1</sup>Quantitative easing is a monetary policy tool in which a central bank focuses on increasing the money supply when standard interest rate targeting is of little use, such as when the funds rate is close to zero. By conducting open-market operations, lending money directly to banks, or purchasing assets from financial institutions, banks are encouraged to lend.

– give rise to credit traps in which monetary policy will be ineffective. In contrast, intermediate levels of asset sales give rise to a quantitative easing, jump-start equilibrium in which monetary policy is effective if pursued sufficiently forcefully. Finally, credit traps will arise when the level of corporate liquidity in the corporate sector at the time of monetary intervention is low. Thus, the model shows that monetary intervention may arrive too late: If liquidity in the corporate sector at the time of intervention is low, monetary expansions will not easily convey additional liquidity from financial intermediaries to firms.

While monetary policy on its own is ineffective in a credit trap equilibrium, our model also shows how *fiscal* policy acting in conjunction with monetary policy can be useful in easing credit traps and increasing bank lending. By circumventing financial intermediaries and directly injecting liquidity into the corporate sector, expansionary fiscal policy can increase collateral values. With collateral values increased, banks can then provide loans to firms. The model therefore points to a natural complementarity between fiscal and monetary policy in stimulating lending.

Finally, since the transmission of monetary shocks does not occur through a neoclassical cost-of-capital effect, the model shows how large changes in aggregate lending and investment can be associated with comparatively small changes in interest rates. This result is consistent with empirical evidence showing that monetary shocks have large real effects even though components of aggregate spending are not very sensitive to cost-of-capital variables (see e.g. Blinder and Maccini 1991). The intuition is that an expansion in monetary policy shifts out both loan supply and loan demand – the latter occurring due to the increase in debt capacity associated with the rise in collateral values. Although the outward shift in loan supply and loan demand increase both lending and investment, they have counteracting effects on the equilibrium interest rate. Small changes in interest rates are therefore coupled with large changes in lending and investment.

The rest of the paper is organized in the following manner. Section 1 provides a brief review of related literature. Section 2 explains the setup of the model. Section 3 analyzes the benchmark case in which liquidation values are determined exogenously. In section 4, which contains the main analysis, we endogenize liquidation values and study their effect on the credit channel transmission of monetary policy. In section 5 we impose more structure on the pricing function of assets by building micro-founded models of collateral values and the market for assets. Section 6 studies the interplay between monetary and fiscal policies in increasing collateral values and boosting bank lending. Section 7 concludes.

## 1. Monetary Policy and the Credit Channel

According to the Credit Channel view of monetary policy transmission, shocks to monetary policy affect the economy through their impact on financial frictions and the availability of credit. This credit view is generally divided into two distinct channels. The first is the ‘balance sheet channel’ in which monetary shocks affect borrower balance sheets. An easing of monetary policy strengthens firms’ balance sheets – for example, by reducing interest rates and raising collateral values – which reduces the cost of external capital and promotes investment and spending. The second channel emphasizes the importance of bank loans to economic activity and is known as the ‘bank lending channel’. According to this view, an expansion of monetary policy shifts the supply of banks loans outwards, and as a result leads to an increase in investment and aggregate demand. Classic studies in the credit channel literature include Bernanke and Blinder (1988), Gertler and Hubbard (1989), Gertler and Gilchrist (1994), Kashyap and Stein (1994), (1995), (2000), Lamont et al. (1994), Bernanke and Gertler (1995), Stein (1998), Holmström and Tirole (1997), and Allen and Gale (2000).

While the credit channel predicts that expansionary monetary policy should lead to increased economic activity, little is known about the limitations of the transmission mechanism of monetary policy in stimulating increased lending. Our paper fills this gap by analyzing how the interplay between liquidity, collateral values and lending can give rise to credit traps. In doing so, the model takes a unified view of the balance sheet and lending channels.<sup>2</sup>

Another strand of literature related to our work is that studying the ongoing financial crisis of 2008 – 2009. This includes Diamond and Rajan (2009), Kashyap, Rajan and Stein (2008), and Shleifer and Vishny (2009) which provide a theoretical framework for the crisis based on the role that securitization played in recent years. Bolton and Freixas (2006) analyzes the role that depleted bank equity capital plays in the transmission of monetary policy in a setting of asymmetric information. Bebchuk and Goldstein (2009) develop a model in which credit market freezes arise as a coordination failure amongst banks lending to firms with interdependent projects. While related to studies of credit crises, our model does not rely on the depletion of bank equity capital. As will be seen, the main friction concerns the lack of liquidity and the strength of balance sheets in the corporate sector. Clearly, though, the addition of features such as bank capital depletion,

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<sup>2</sup>Diamond and Rajan (2006) present a model unifying the traditional ‘money channel’ view with the ‘balance sheet channel’, and Holmström and Tirole (1997) identifies both ‘balance sheet’ and ‘lending’ channels.

uncertainty regarding the strength of bank balance sheets, and debt-overhang in bank financing – all of which played an important role during the financial crisis of 2008-2009 – will only serve to further hinder the transmission of monetary policy.

Our paper is also related to numerous studies on credit cyclicalities and the financial accelerator. In this literature, pioneered in Bernanke and Gertler (1989), countercyclical frictions in the cost of external finance, driven by pro-cyclical variation in the strength of firms' balance sheets, serve to amplify the business cycle. Important studies in this field include Shleifer and Vishny (1992), Kiyotaki and Moore (1997), Holmström and Tirole (1997), and Fostel and Geanakoplos (2008).

Finally, as described above, our work is closely related to Shleifer and Vishny (1992) which first introduces the positive feedback loop between liquidity and collateral values, debt capacity, and the provision of credit. Other recent papers which study the interplay between liquidity, fire sales, and asset prices are Acharya and Viswanathan (2009), Acharya, Shin and Yorulmazer (2009), and Rampini and Viswanathan (2009). Our analysis is also related to Holmström and Tirole (1997) which analyzes how the distribution of wealth across firms and suppliers of capital affects lending and investment. Holmström and Tirole, however, consider exogenous asset values while we endogenize these values and analyze their interplay with liquidity and lending.<sup>3</sup>

## 2. Model Setup

Consider an economy comprised of a continuous set of firms with measure normalized to unity, a set of commercial banks which can supply capital to firms, and a central bank. The firms in our model are each endowed with an identical opportunity to invest in a project. The project requires an initial outlay of  $I$  at date-0, and returns a cash flow of  $X_1$  in date-1 and  $X_2$  in date 2. As in Hart and Moore (1998) cash flows are assumed to be unverifiable. For simplicity we assume that  $I < X_1 < X_2$ .<sup>4</sup> If undertaken, a project can be liquidated at date-1 for a value denoted by  $L$ . The liquidation value of assets will play a key role in the analysis and will be described further below.

Firms differ in their level of internal wealth,  $A$ , with  $A$  distributed over the support  $[0, I]$ . For convenience, firms are parameterized by the level of borrowing that they require in order to invest in the project  $B = I - A$ . We assume that  $B$  is distributed according to the cumulative distribution

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<sup>3</sup>Indeed, according to Holmström and Tirole (1997): ‘A proper investigation of the transmission mechanism of real and monetary shocks must take into account the feedback from interest rates to capital values.’

<sup>4</sup>While by no means necessary, this assumption eases exposition and is consistent with our main interest of tight liquidity in date-1.



function  $G()$ , where for simplicity  $G$  is twice differentiable.

To invest in their project, firms can borrow capital from banks. We assume that firms cannot issue bonds in the capital markets. While this is a strong assumption, adding a bond market does not change our results qualitatively, as long as banks are assumed to have some informational or monitoring advantage in providing capital.<sup>5</sup>

As is common in the literature on the lending channel of monetary policy (see, e.g. Kashyap and Stein, 1994) we assume, for simplicity, that the supply of loanable funds,  $R$ , is directly determined by the central bank.<sup>6</sup> This can be thought of as occurring in a number of ways. First, as in the lending channel literature, open market operations can shift loan supply by influencing the level of reserves and hence of deposits.<sup>7</sup> Another method by which the central bank can influence loan supply is through direct loans to the banking sector.<sup>8</sup> Finally, in extreme cases – as in the financial crisis of 2008-2009 – shifts in loan supply can be brought about through government equity injections to banks.

To conclude the setting, we assume that both banks, as well as firms, can invest in a security yielding a return normalized to zero rather than engaging in lending or borrowing. One can think of this security as investment in government debt.<sup>9</sup>

While most of our predictions stem from a general equilibrium analysis in which we endogenize the liquidation value of assets, it is useful to begin the analysis with the benchmark case of exogenous liquidation values.

### 3. The Benchmark Case: Exogenous Liquidation Values

We begin by assuming that the liquidation value of the project  $L$  is exogenously determined. As we show in the next section, we can restrict our attention to cases in which  $L$  is smaller than  $X_1$ ,

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<sup>5</sup>That intermediated loans are somehow ‘special’ is a fundamental assumption in the lending channel literature (see Bernanke and Blinder (1988)).

<sup>6</sup>The central bank is exogenous to the model – its sole role is in influencing  $R$  – and so it is not assigned an objective function. In addition, to obtain monetary non-neutrality, we make the standard assumption of imperfect price adjustment.

<sup>7</sup>This implicitly assumes that there are frictions in banks’ ability to insulate lending from shocks to reserves by switching to other forms of non-reservable finance such as equity, commercial paper, or long-term debt. For a discussion see Kashyap and Stein (1995), Stein (1998).

<sup>8</sup>The Federal Reserve used this method during the crisis of 2008-2009 under the Term Auction Facility. Indeed, it is argued that by expanding the set of acceptable forms of collateral, the Federal Reserve was actually providing subsidized loans to the banking sector, and hence was in effect recapitalizing banks.

<sup>9</sup>The interest rate provided by government debt can be endogenized to depend on the level of demand for such debt by both the banking and corporate sector. Doing so would not change our main results.

since once  $L$  is endogenized this inequality holds in equilibrium. Further, we consider the more interesting case where  $L < I$ .<sup>10</sup>

Consider a firm which needs to borrow an amount  $B$  to invest in its project and is faced with an interest rate  $r$ . Since cash flow is unverifiable, there is no way to induce the firm to repay at date 2. As is common in the literature in incomplete financial contracts, the only method to induce the firm to repay at date-1 is through the threat of liquidation (see, for example, Hart and Moore (1994)). Assuming that at date-1 the firm has all the bargaining power in renegotiating its debt obligation with its bank, the firm will never be able to commit to repay more than  $L$  at date-1 as it can always bargain down its repayment to the bank's outside option. Thus, the firm will be able to borrow an amount  $B$  only when  $B(1+r) \leq L$ , or equivalently, when

$$B \leq \frac{L}{1+r}. \quad (1)$$

Faced with an interest rate  $r$ , a firm will choose to borrow  $B$  and invest in its project rather than invest its internal funds  $A$  in the zero-interest security when  $X_1 + X_2 - (1+r)B \geq A$ .<sup>11</sup> Equivalently, since  $B = I - A$ , this occurs when

$$B \leq \frac{X_1 + X_2 - I}{r}. \quad (2)$$

Inequality (2) represents the participation constraints of firms and is driven by the cash flows generated by the project and their initial financial constraints. Combining (1) and (2) yields that at an interest rate  $r$ , all firms with borrowing requirement

$$B \leq \min\left[\frac{L}{1+r}, \frac{X_1 + X_2 - I}{r}\right] \quad (3)$$

are both able and willing to borrow funds to invest in their respective projects. At any interest rate  $r$ , the demand for capital generated by firms is therefore given by:

$$D(r) = \int_0^{B^*(r)} B dG(B), \quad (4)$$

where  $B^*(r) = \min[L/(1+r), (X_1 + X_2 - I)/r]$  represents the marginal firm that borrows and invests in the project as a function of the interest rate  $r$ . As can be seen, the liquidation value of

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<sup>10</sup>When  $L > I$  the analysis continues to hold but the financial frictions are negligible since liquidation of the project at the end of the first period would yield enough to fully repay the bank.

<sup>11</sup>Note that firms invest non-utilized capital in the zero-interest security. At the cost of ease of exposition, we could also assume that firms invest non-utilized capital in banks.

assets,  $L$ , thus plays a role in determining demand for loanable funds through its impact on financial constraints. Indeed, for low enough  $r$  inequality (1) binds while inequality (2) does not: demand for loanable funds is determined by firms' ability to borrow (as constrained by liquidation values) rather than their desire to borrow (as determined by the participation constraint). To emphasize this, we refer to the demand function in (4) as 'effective demand', thereby differentiating it from the demand that would have been obtained under no financial frictions.

Equilibrium in the model is determined by equating effective demand for loanable funds to the supply of loanable funds:

$$\int_0^{B^*(r)} BdG(B) \leq R, \text{ with strict inequality only when } r = 0 \quad (5)$$

From (5) it is easy to see that as the central bank increases the supply of funds, the interest rate decreases and aggregate lending increases. Importantly, however, the liquidation value  $L$  will determine the maximal level of aggregate lending. At a zero interest rate, financial frictions imply that the maximal amount a firm can borrow is  $B = L$ . Thus, as can be seen from (4), for any exogenous  $L$  the maximal effective demand is obtained at  $r = 0$  and equals  $\int_0^L BdG(B)$ . From (5), any increase by the central bank of loan supply beyond  $\int_0^L BdG(B)$  will not increase lending to the corporate sector, but will instead be invested by banks in the zero-interest security. Aggregate lending from banks to firms therefore increases one-to-one with the loan supply  $R$ , up to the point  $R = \int_0^L BdG(B)$ , after which it remains constant.

In sum, the liquidation value of assets limits the effectiveness of monetary policy. Monetary policy itself, however, *shifts* liquidation values through its effect on lending and corporate liquidity. Thus, to understand the limits of the transmission mechanism of monetary policy, it is crucial to endogenize the interplay between lending, liquidity and liquidation values.

## 4. The Credit Channel with Endogenous Liquidation Values

To endogenize liquidation values, we assume that when a bank repossess the assets of a firm which has defaulted it must sell these assets instead of operating the asset itself. The value obtained in this sale is the liquidation value of assets. Following Shleifer and Vishny (1992), we assume that the best users of a defaulted firm's assets are other firms within the same industry. Industry participants bid for the defaulted firm's assets, so that demand will be determined both by the potential value of the assets as well as the liquidity constraints of the bidders. As in Shleifer and

Vishny (1992), if the liquidity available to the bidders is sufficiently low, the value obtained for the asset will be lower than its first-best value.<sup>12</sup>

Before continuing, it is useful to provide a general description of the model's main effects. The model combines the 'balance-sheet channel' and the 'lending channel' in a general equilibrium rational expectation framework. This can be described with the following series of interlocking forces. When the central bank eases monetary policy, the supply of loanable funds increases. Similar to a standard 'lending channel' effect (see e.g. Kashyap and Stein (1995)), banks will tend to lend out more funds, which will increase liquidity in the corporate sector. As liquidity in the corporate sector increases, liquidation value of assets will increase – firms become less liquidity constrained, and can hence bid more aggressively when acquiring assets of liquidated firms. As in a standard 'balance sheet channel' effect (e.g. Bernanke and Gertler (1989, 1990, 1995), Lamont (1995)), this endogenous increase in liquidation values improves firms' collateral positions, which enhances their ability to borrow the additional liquidity which was injected to the commercial banks by the central bank.

In equilibrium, the lending and balance sheet channels of monetary policy are therefore linked through endogenous liquidation values: increased bank lending leads to greater liquidity in the corporate sector and thus higher collateral prices, while higher collateral prices reduces financial frictions and enables banks to increase lending to firms.

Initially, rather than imposing a particular structure on the market for repossessed assets, we analyze the results using a general specification where the price of assets in liquidation depend on the level of liquidity in the corporate sector and its distribution.<sup>13</sup> Accordingly, we define a pricing function,  $P$ , for the liquidation value of assets that takes as inputs two variables which jointly span the level and distribution of liquidity at date-1 within the corporate sector. The first variable is  $B^*$ , the marginal firm that successfully obtained funding at date-0. The second variable is the equilibrium interest rate  $r^*$  paid by firms borrowing at date-0.<sup>14</sup> Thus, if a firm defaults and its assets are repossessed by a bank and sold on the market, the price of these assets will be

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<sup>12</sup>Empirical evidence for this industry equilibrium model and its implications for liquidation values, corporate liquidity and debt financing is provided in Benmelech (2009), Benmelech and Bergman (2009) and Pulvino (1998).

<sup>13</sup>In Section 5 we impose more structure on the pricing function by modeling the market for assets in two ways: bargaining and a competitive market.

<sup>14</sup>Other exogenous determinants of the date-1 distribution of liquidity is the date-0 distribution of internal funds,  $G$ , and the level of date-1 cash flows  $X_1$ . In this section, we suppress in our notation of the pricing function its dependency on  $G$  and  $X_1$ . In Section 6, we consider how the pricing function varies with these exogenous variables. For simplicity, we assume that  $P$  is differentiable in  $B^*$ ,  $r^*$ , and  $X_1$ .

$P = P(B^*, r^*)$ . For simplicity, we assume that all assets of a firm are essential in generating cash flow, which implies that partial liquidation of assets is useless. This implies that if a firm defaults and its assets are repossessed by its bank and sold on the market, the maximal price of these assets will be  $X_1$ , the maximal amount of cash holdings of any potential buying firm.

We make the reasonable assumption that if date-1 corporate liquidity increases, the price of liquidated assets does not go down. This implies:

**Liquidity Pricing Assumption.**

(i)  $\partial P / \partial B^* \geq 0$

(ii)  $\partial P / \partial r^* \leq 0$

These assumptions are straightforward. First, as the proportion of firms obtaining funding at date-0 increases, date-1 liquidity increases, as does, therefore, the price of liquidated assets.<sup>15</sup> The pricing function will therefore be increasing in  $B^*$ , the marginal firm obtaining finance. Similarly, as the interest rate at which firms borrow increases, date-1 liquidity decreases, so that  $P$  will be decreasing in  $r^*$ .

#### 4.1. Equilibria with Endogenous Liquidation Values

Given a pricing function  $P$ , an equilibrium in the lending market is characterized as follows:

**Market Equilibrium.** *An equilibrium in the lending market is a vector  $\{R, r^*, L^*, B^*\}$ , such that:*

- (i) *Firms optimize in their borrowing and investing choices given the interest rate  $r^*$  and the liquidation value of assets  $L^*$ .*
- (ii) *Banks optimize in their lending choices, knowing that firms can commit to repay no more than  $L^*$ .*
- (iii) *The market for loanable funds clears at date-0: Denoting by  $B^*$  the marginal firm which borrows to invest in a project, the market clearing condition is*

$$\int_0^{B^*(r)} B dG(B) \leq R, \text{ with strict inequality only when } r^* = 0$$

- (iv)  *$L^*$  is an equilibrium liquidation value:  $L^* = P(B^*, r^*)$ .*

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<sup>15</sup>Throughout the paper, all monotonicity statements refer to weak monotonicity unless stated otherwise.

The equilibrium requirements are quite intuitive. First, in equilibrium firms will optimize their borrowing choices. Since each individual firm takes the liquidation value  $L^*$  as exogenous, this requirement translates into the optimality condition developed in inequality (3) of the previous section: a firm with borrowing requirement  $B$  borrows if and only if  $B \leq \min[\frac{L^*}{(1+r^*)}, \frac{(X_1+X_2-I)}{r^*}]$ .

In optimizing lending decisions, banks will lend at the equilibrium interest rate  $r^*$  while understanding that firms cannot commit to repay more than  $L^*$ . Further, in equilibrium, for any rate  $r^* > 0$  realized demand for loanable funds will equal supply. In contrast, when  $r^* = 0$  the supply of loanable funds can be greater than the demand – any excess supply will simply be invested by the banks in the zero-interest security.<sup>16</sup>

Finally, equilibrium requirement (iv) is a rational expectations condition, stating that the liquidation value of assets taken as given by individual banks when making their date-0 decisions is indeed the date-1 price of liquidated assets. As described above, this price is determined through a Shleifer-Vishny (1992) equilibrium by the liquidity in the corporate sector and is governed by the pricing function  $P$ . It should be noted that since there is no uncertainty about project outcomes there will be no liquidation on the equilibrium path. As in Hart and Moore (1994), all threats of default are strategic rather than liquidity driven. To the extent that a strategic default is credible – i.e., if the promised payment is greater than the liquidation value,  $L$  – debt renegotiation ensues. As is standard in the financial contracting literature, if initial contracts are renegotiation proof, there will therefore be no on-the-equilibrium path reductions in debt payments.

We solve for the equilibrium in the following manner. First, the analysis of exogenous liquidation values in Section 2 shows that for every potential liquidation value  $L$  and loan supply  $R$ , there exist an associated equilibrium interest rate  $r^*$  and an equilibrium marginal borrowing firm  $B^* = \min[\frac{L}{(1+r^*)}, \frac{(X_1+X_2-I)}{r^*}]$ . We can thus define for any liquidation value  $L$  and loan supply  $R$  the associated equilibrium interest rate and marginal borrowing firm,  $r^*(L; R)$  and  $B^*(L; R)$ . Using these, we define for every direct pricing function  $P(B, r)$  an *indirect* pricing function

$$p(L; R) \equiv P(B^*(L; R), r^*(L; R)), \quad (6)$$

which takes as input the liquidation value  $L$  and the exogenously given loan supply  $R$  and provides as output the implied price of assets given  $L$  and  $R$ .

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<sup>16</sup>Note that the assumption that banks cannot raise external finance implies that no bank will be able to reduce the interest rate it offers to increase loan capacity and profits. Essentially, banks' marginal cost of raising funds beyond the reserves they have is assumed to be infinite. More generally, as in a standard lending channel framework, all that is required is non-zero marginal costs in raising non-reservable forms of liabilities.

It is then easy to see that for the rational expectations equilibrium condition (iv) to be satisfied, the equilibrium liquidation value  $L^*$  must be a fixed point of  $p$  that satisfies  $p(L^*; R) = L^*$ . If banks at date-0 lend capital under the assumption that the date-1 liquidation value of assets will be  $L^*$ , then at date-1, the price of liquidated assets, as determined by the amount of liquidity in the corporate sector in date-1 should indeed be  $L^*$ . Formally, we have the following proposition:

**Proposition 1.** *Assume an exogenous supply of loans  $R$ . Then  $L^*$  is an equilibrium liquidation value if and only if*

$$p(L^*; R) = L^*. \quad (7)$$

The equilibrium interest rate is then given by  $r^*(L^*; R)$ , while the marginal firm that borrows in this equilibrium is given by  $B^*(L^*; R)$ .

*Proof.* See Appendix.

To characterize the pricing function  $p(L; R)$ , it is useful to define for every amount of loanable funds,  $R \leq \int_0^I B dG(B)$ , the value  $\bar{B}(R)$  which represents the marginal firm that obtains financing assuming that the *full* amount  $R$  is lent out by banks.<sup>17</sup> It is easy to see that  $\bar{B}(R)$  is given implicitly by the equation:

$$\int_0^{\bar{B}(R)} B dG(B) = R. \quad (8)$$

The indirect pricing function  $p(L; R)$  is then characterized by the following proposition.

**Proposition 2.** *Fix an exogenous liquidation value of assets  $L$  and loan supply  $R \leq \int_0^I B dG(B)$ .*

(1) *For any  $L < \bar{B}(R)$ :*

(i) *The equilibrium interest rate associated with the pair  $(L, R)$  will be  $r^* = 0$ , and the marginal firm able to borrow will have a borrowing requirement of  $B^* = L$ .*

(ii) *The indirect pricing function therefore satisfies  $p(L; R) = P(L, 0)$ .*

(iii) *Demand for loanable funds,  $\int_0^L B dG(B)$  will be smaller than the supply  $R$ , implying that not all of the supply will be lent out.*

(2) *For any  $L \geq \bar{B}(R)$ :*

(i) *The market for loanable funds clears, with the entire loan supply lent out.*

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<sup>17</sup>  $R_{max} = \int_0^I B dG(B)$  is the maximal level of aggregate lending possible in the economy.

(ii) The equilibrium interest rate will be such that the marginal borrowing firm will have a borrowing requirement of  $\bar{B}(R)$ . In equilibrium, therefore,  $B^* = \bar{B}(R)$  and  $r^*$  satisfies  $\bar{B}(R) = \min[\frac{L}{(1+r^*)}, \frac{(X_1+X_2-I)}{r^*}]$ .

(iii) The indirect pricing function satisfies  $p(L; R) = P(\bar{B}(R), r^*)$ , with  $r^*$  given in 2(ii).

*Proof.* See Appendix.

To understand Proposition 2 consider first a potential equilibrium liquidation value  $L$  satisfying  $L < \bar{B}(R)$ . Since at a zero interest rate the maximal amount firms can borrow is  $L$ , realized demand at  $r = 0$  is  $\int_0^L B dG(B)$ . Since, by assumption,  $L$  is smaller than  $\bar{B}(R)$ , realized demand at a zero interest rate will be smaller than  $\int_0^{\bar{B}(R)} B dG(B) = R$ , the supply of loanable funds. Because equilibrium interest rates cannot fall below zero, the equilibrium interest rate associated with any  $L$  smaller than  $\bar{B}(R)$  will indeed be zero and the associated marginal borrowing firm will have  $B = L$ . By definition, the pricing function will satisfy  $p(L; R) = P(L, 0)$  on the region  $L \leq \bar{B}(R)$ .<sup>18</sup> Further, in equilibrium, in this region not all of the loan supply will be lent out: realized aggregate lending,  $\int_0^L B dG(B)$ , will be smaller than loan supply,  $R$ .

Consider now a potential equilibrium liquidation value  $L$  satisfying  $L > \bar{B}(R)$ . In this case, following the lines of the argument above, we have that realized demand at a zero interest rate,  $\int_0^{\bar{B}(R)} B dG(B)$ , is greater than the supply of loanable funds,  $R$ . Thus, in equilibrium, the interest rate will shift upwards so as to equate the supply and demand for loanable funds.<sup>19</sup> Put differently, the equilibrium interest rate will be set such that the marginal firm will have borrowing requirement  $\bar{B}(R)$ , thereby guaranteeing that all of  $R$  is lent out.

A direct consequence of Proposition 2 which we use in the next section is:

**Corollary 1.** Fix an exogenous loan supply  $R \leq \int_0^I B dG(B)$ . The indirect pricing function  $p(L; R)$  is increasing in  $L$  over the region  $L < \bar{B}(R)$  and decreasing in  $L$  over the region  $L > \bar{B}(R)$ .

Holding  $R$  constant, increasing  $L$  has two opposing effects on the price of collateral in date 1. The first effect is that as  $L$  increases, more firms are able to raise external finance which increases liquidity in the corporate sector and therefore raises the market price of date-1 collateral. The second effect is that as  $L$  increases, more firms are able to borrow. Effective demand for intermediated loans increases, which implies that the equilibrium interest rate of loans rises. An increase in the interest rate reduces liquidity in the corporate sector in date 1, which tends to push

<sup>18</sup>Recall that  $P$  is the direct pricing function.

<sup>19</sup>In the knife-edge case where  $L = \bar{B}(R)$ ,  $r = 0$  will be an equilibrium interest rate.



down the date-1 price of collateral.<sup>20</sup> When  $L$  is low the first effect dominates, while when it is high the second dominates.  $p(L; R)$  is therefore non-monotonic in  $L$ .

Combining Proposition 2 with Corollary 1 shows how shifts in monetary policy influence the indirect pricing function. This is illustrated in Figure 1 which presents the impact of an increase in the supply of funds from  $R_1$  to  $R_2$ . By Proposition 2, the pricing function  $p(L; R)$  is identical to the function  $P(L, 0)$  up to the point  $\bar{B}(R)$ , after which for any  $L > \bar{B}(R)$  it is decreasing. As can be seen in the figure,  $P(L, 0)$  therefore serves as an envelope of  $p(L; R)$ : for any  $R$ , the two functions are equal up to the point  $\bar{B}(R)$ , while  $P(L, 0)$  is greater than  $p(L; R)$  for  $L$  greater than  $\bar{B}(R)$ .

## 4.2. Monetary Policy, Liquidation Values, and Lending

In this section we characterize the impact of monetary policy on lending, collateral values, and interest rates when the value of assets is determined endogenously. We will say that monetary policy is ‘effective at  $R$ ’ if, in equilibrium, when loan supply is equal to  $R$ , the entire loan supply is lent out. Alternatively, monetary policy is ‘ineffective at  $R$ ’ if when loan supply is equal to  $R$ , an amount strictly less than  $R$  is lent out in equilibrium. We begin with the following proposition.

**Proposition 3.** *Consider a loan supply  $R$ . Banks can lend out  $R$  in loans in equilibrium if and only if  $P(\bar{B}(R), 0) \geq \bar{B}(R)$ . Consequently, monetary policy is effective at a loan supply  $R = \bar{B}^{-1}(L)$  if and only if  $P(L, 0) \geq L$ .*

Proposition 3 is quite intuitive. First, the *minimal* level of collateral required to extract  $R$  in loans is  $\bar{B}(R)$ . To see this note that if the value of collateral,  $L$ , is less than  $\bar{B}(R)$ , since no firm can borrow more than  $L$ , the marginal firm able to borrow will have borrowing requirement  $B < \bar{B}(R)$ . By definition of  $\bar{B}(R)$ , this then implies that aggregate loan supply lent out will be smaller than  $R$ . Second, the *maximal* value of collateral when  $R$  is lent out is  $P(\bar{B}(R), 0)$ . This is because when  $R$  is lent out the marginal firm obtaining financing will have a borrowing requirement of  $\bar{B}(R)$ , and liquidity is highest when the interest rate is zero. Proposition 3 states that if the maximal value of collateral conditional on  $R$  being lent out – i.e.  $P(\bar{B}(R), 0)$  – is smaller than the minimal amount of collateral required to extract  $R$  – i.e.  $\bar{B}(R)$  – then  $R$  will not be lent out. In contrast,  $R$  will be lent out if the maximal value of collateral associated with  $R$  being lent out is greater than the

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<sup>20</sup>This effect is similar to the analysis of Diamond and Rajan (2001) which shows that an adverse effect of liquidity provision is to raise real interest rates which may lead to more bank failures and lower subsequent aggregate liquidity.

minimal value of collateral required to extract  $R$ . In this case, the equilibrium interest rate and liquidation value will adjust to equate effective loan demand to loan supply,  $R$ .

Using Proposition 2 and Proposition 3, we can analyze the general equilibrium effects of shifts in the supply of loanable funds. Proposition 4 provides a formal characterization of three types of equilibria that arise.

**Proposition 4.** *Consider the pricing function  $P(L, 0)$ .*

(i) **The conventional equilibrium:** If  $P(L, 0) > L$  for all  $0 \leq L \leq I$  then aggregate lending increases one-for-one with increases in the loan supply  $R$  on the range  $0 \leq R \leq R_{max}$ , where  $R_{max} = \int_0^I BdG(B)$  is the maximal level possible of aggregate lending. Monetary policy is therefore effective at any level of reserves  $R \leq R_{max}$ . Further, in this region the equilibrium liquidation value of assets is increasing in loan supply  $R$ .

(ii) **The credit trap equilibrium:** Assume that  $P(L, 0)$  is concave in  $L$ , with  $P(L^*, 0) = L^*$ ,  $P(L, 0) > L$  for  $0 \leq L < L^*$ , and  $P(L, 0) < L$  for  $L > L^*$ . Then monetary policy is effective up to the loan supply  $R^* = \bar{B}^{-1}(L^*)$  and ineffective beyond  $R^*$ . Increases in loan supply beyond  $R^*$  do not increase lending, nor do they change the equilibrium liquidation value of assets which remains constant at  $L^*$ . Maximal aggregate lending is therefore  $R^*$  and the maximal liquidation value of assets is  $L^*$ .

(iii) **The jump-start equilibrium:** Assume that  $P(L, 0)$  is convex in  $L$  over the region  $[L_1, L_2]$ , with  $P(L_i, 0) = L_i$  for  $i=1,2$ ,  $P(L, 0) > L$  for  $0 \leq L < L_1$ , and  $P(L, 0) < L$  for  $L_1 < L < L_2$ . Then, monetary policy is ineffective over the region  $(R_1, R_2)$ , where  $R_i = \bar{B}^{-1}(L_i)$  ( $i=1,2$ ), but is effective at loan supply  $R_2$ . Further, increases in loan supply over the region  $[R_1, R_2]$  do not change the equilibrium liquidation value which remains constant at  $L_1$ . However,  $L_2$  is an equilibrium liquidation value at loan supply  $R_2$ .

*Proof.* In the Appendix.

To understand Proposition 4 we consider each of the three equilibrium types in turn.

## Conventional Equilibrium

First, consider the conventional equilibrium in case (i). Since  $P(L, 0) > L$  for all  $0 \leq L \leq I$ , Proposition 3 directly implies that any loan supply  $R$  can be lent out, up to the maximal possible level of lending,  $R_{max} = \int_0^I B dG(B)$ . Monetary policy in this equilibrium is therefore fully effective: increases in  $R$  are matched one to one with increases in aggregate lending, up to  $R_{max}$ .

Figure 2 demonstrates this conventional equilibrium. Increases in  $R$  shift out the indirect pricing function  $p(L; R)$  as described in Proposition 2. This shift in  $p(L; R)$  implies that the equilibrium liquidation value – i.e. the price of assets – will increase (from  $L_1^*$  to  $L_2^*$  in the figure). The overall chain of events of an increase in loan supply can then be summarized as follows: The increased loan supply is lent out to the corporate sector; the increased liquidity in the corporate sector increases collateral values; and finally, the increase in collateral values increases firm debt capacity, thereby enabling the increase in loan supply.

The effect on equilibrium interest rates of a shift in loan supply is less clear cut. This is best demonstrated in Figure 3 which graphs loan supply and loan demand as a function of interest rates,  $r$ . The main point is that *effective* demand for loans is not just a function of the loan interest rate, but is also influenced by collateral values. Firm borrowing in the model is determined both by their desire to borrow as well as their ability to do so. Loan demand can thus be represented by the function  $D(r; L^*)$ , where  $L^*$  is the equilibrium collateral price.

As can be seen in Figure 3, an increase in  $R$  has two effects. First, loan supply shifts out. Second, effective loan demand shifts out as well – equilibrium liquidation values increase, thereby increasing firm borrowing capacity. While the outward shifts in loan supply and loan demand both push aggregate lending upwards, they have countervailing effects on the equilibrium interest rate. If loan demand shifts out sufficiently – due to a large increase in collateral prices – the change in the equilibrium interest rate will be small.<sup>21</sup> Put differently, in a conventional equilibrium large changes in aggregate lending and investment can be associated with small changes in interest rates. This is consistent with evidence that monetary shocks have large real effects, even though empirical studies show that components of aggregate spending are not very sensitive to cost-of-capital variables (see e.g. Blinder and Maccini(1991)).

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<sup>21</sup>In fact, the equilibrium interest rate may actually rise with increases in loan supply. Formally, it is easy to show that the condition for this is  $\frac{\partial P(\bar{B}(R), r^*)}{\partial B^*} > 1 + r^*$ . That is, the sensitivity of the value of collateral to changes in liquidity (as proxied by  $B^*$ , the marginal firm obtaining financing) is sufficiently large. We return to this point when discussing jump-start, quantitative easing equilibria.

### Credit Trap Equilibrium

Consider now the credit trap equilibrium represented in case (ii) of Proposition 4. In this equilibrium, monetary policy is ineffective at any point beyond  $R^*$ . The intuition is that for any additional loan supply above  $R^*$  to actually be lent out by banks, liquidation values need to be sufficiently high. However, in a credit trap the implied increase in date-1 liquidation values associated with a marginal increase aggregate lending above and beyond  $R^*$  is not sufficient to induce banks to actually lend the additional funds at date-0. Monetary policy thus becomes ineffective above the loan supply  $R^*$ .

The equilibrium is depicted in Figure 4. If the central bank sets reserve level at  $R_1 < R^*$ , there is a positive equilibrium liquidation value –  $L_1$  in the figure – in which aggregate lending is  $R_1$ . However, monetary policy is completely ineffective at any point beyond  $R^*$ . As loan supply increases beyond  $R^*$ , say to  $R_2$  in the figure, the sole positive equilibrium liquidation value remains at  $L^*$  and equilibrium lending remains constant at  $R^*$ . This is because the rate at which the implied value of collateral increases is not sufficiently high to enable banks to lend the additional loan supply. Put differently, banks rationally understand that lending any incremental amount beyond  $R^*$  does not increase collateral values sufficiently to support the additional lending. Since the equilibrium liquidation value  $L^*$  does not change with increases in loan supply above  $R^*$ , firm borrowing capacity remains constant. This implies that realized lending remains at  $R^*$  with the difference  $R - R^*$  invested by banks. Finally, since beyond loan supply  $R^*$  effective loan demand is smaller than loan supply, based on Theorem 2(i), the equilibrium interest rate will remain constant at zero. Monetary policy is powerless in increasing lending, collateral values or corporate liquidity.

To emphasize, note that in this credit trap equilibrium, increased liquidity in the corporate sector would have increased collateral values which could then serve to enable additional lending. The issue, though, is that banks are not willing to supply the additional liquidity on their own. Regardless of the stance of monetary policy, collateral values therefore remain depressed at a low level implied by the lack of liquidity in the corporate sector.<sup>22</sup>

### Jump-start equilibrium

Consider now the jump-start equilibrium of case (iii) in Proposition 4. As exhibited in Figure 5,

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<sup>22</sup>This reasoning suggests that direct injections of liquidity into the corporate sector may be useful – a point we return to in Section 6.

for any loan supply  $R_1 < R < R_2$ , the only equilibrium has a liquidation value of  $L = L_1$  and associated lending of  $R_1 = \bar{B}^{-1}(L_1)$ .<sup>23</sup> Increases in the loan supply over the region  $[R_1, R_2)$  are therefore completely ineffective in increasing lending and collateral values. There is no response to injections of liquidity by the central bank: the equilibrium liquidation value is stuck at  $L_1$  and lending remains constant  $R_1$ . Further, since as in a credit trap equilibrium, effective loan demand will be depressed due to the low level of liquidation values, in this region the interest rate on loans will be constant at zero, its lower bound.

In contrast, if the central bank acts forcefully enough by increasing loan supply to  $R_2$ , another equilibrium arises. In this equilibrium the liquidation value of assets is high –  $L_2$  in the figure – enabling the full loan supply  $R_2$  to be lent out. This equilibrium arises due to the feedback effect between lending and collateral values: lending is high because collateral values are large enough to support it, while collateral values are high because lending increases liquidity in the corporate sector.

The jump-start equilibrium therefore demonstrates how a policy of quantitative easing may successfully reignite bank lending. Banks know that when loan supply is moderate, the implied value of collateral conditional on lending occurring is not high enough to actually justify lending. However, when loan supply is expanded sufficiently, a new high lending and high collateral value rational expectations equilibrium arises.

It should be emphasized that although a new equilibrium arises with a sufficiently forceful monetary expansion, it is by no means clear that banks will successfully coordinate on it. If each bank assumes that the others will continue lending at depressed levels, the economy will be stuck in the inefficient equilibrium. In this sense, a policy of quantitative easing in and of itself may not be sufficient to jump-start lending and collateral values. The central bank, or more generally government at large, may require other tools to solve the coordination problem arising between banks.<sup>24</sup>

The jump-start equilibrium also explains how small contractions in the stance of monetary policy can lead to large crashes in asset values and lending. This is simply the flip-side of quantitative easing. Returning to Figure 5, consider an economy with loan supply at  $R_2$  and liquidation value

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<sup>23</sup>To see this note that for any  $R$  in this region, the only value where  $p(L; R)$  equals  $L$  is  $L = L_1$ . Further, in this region  $P(\bar{B}(R), 0) < \bar{B}(R)$  which by Proposition 3 implies that  $R$  will not be lent out.

<sup>24</sup>To eliminate the low lending equilibrium, these actions may include government subsidies for new loans, a tax on bank reserves, or government prodding to increase lending (as seen, for example, during the crisis of 2008-2009).

at  $L_2$ . An incremental reduction in loan supply from  $R_2$  leads to a collapse in lending and collateral values to  $R_1$  and  $L_1$ , respectively. The small reduction in lending reduces liquidity and collateral values, which in turn reduces liquidity further. The negative feedback effect between lending, liquidity and collateral values drives all three to lower levels until the process stops at the new equilibrium.

Interestingly, the monetary contraction will also *reduce* the interest rates (to zero according to Proposition 2). The intuition is that while loan supply decreases by a small amount, effective loan demand collapses because of the attendant drop in collateral values. A form of flight to quality arises in which the supply of loans is distributed at low cost to the comparatively small number of firms that have balance sheets strong enough to borrow.

Taken together, therefore, in this equilibrium monetary contraction can lead to crashes in lending and collateral values coupled with a reduction in interest rates. These effects are very much consistent with accounts of the Japanese experience during the 1980s such as Bernanke and Gertler (1995) who argue that “the crash of Japanese land and equity values in the latter 1980s was the result (at least in part) of monetary tightening; ... [T]his collapse in asset values reduced the creditworthiness of many Japanese corporations and banks, contributing to the ensuing recession.”

To conclude, Proposition 4 shows that the efficacy of monetary policy crucially depends on the shape and level of the pricing function  $P$  – i.e. on how collateral prices vary with corporate liquidity. A low and concave pricing function will generally lead to a credit trap equilibrium where quantitative easing will be ineffective in increasing lending and liquidation values, regardless of the level of loan supply set by the central bank. In contrast, convexities in the pricing function will lead to a jump start equilibrium where quantitative easing may be useful. What determines the shape of the pricing function – and hence the efficacy of quantitative easing – is therefore a natural question which we turn to in the next section.

## 5. Microfoundations of the Collateral Pricing Function

In this section we impose more structure on the pricing function of assets,  $P$ , by building micro-founded models of collateral values and the market for assets. In the first model, collateral values are determined in a competitive market for repossessed assets. The second model involves a bargaining setting in which, when selling repossessed assets, the bank negotiates a price with a limited

set of potential buyers. Finally, we show that search costs associated with finding a suitable buyer for assets introduces convexities in the pricing function and hence the possibility of a jump-start, quantitative easing equilibrium.

## A Competitive Market for Repossessed Assets

Consider the case where the price of collateral is determined in a competitive market in which the equilibrium price of assets at date-1 is set so as to equate the demand and supply of assets. We assume that firms that invested at date zero – which we call industry insiders – develop the know-how to operate assets of firms liquidated at date 1. Operating liquidated assets enables these firms to generate cash flow  $V$  with  $V < X_2$  at date 2.<sup>25</sup> For ease of exposition, we further assume that  $V > X_1$ . Since  $X_1$  is the maximal wealth of firms in the economy, this assumption implies that at date-1 firms will be willing to pay their full internal wealth to obtain liquidated assets.<sup>26</sup>

Aside from demand stemming from industry insiders, we assume that there is a perfectly elastic supply of capital at date-1 ready to buy liquidated assets at a positive price of  $kV < X_1$ . This supply of capital should be thought of as stemming from industry outsiders who can employ the assets but only in an inferior use.

Finally, we assume that at date-1, a supply  $\alpha$  of assets, exogenous to the model, are sold on the market. Variations in  $\alpha$  capture shifts in the supply of assets which must be cleared in the market.<sup>27</sup> For example, in a downturn the supply of assets sold on the market will be high.

To obtain the pricing function  $P(L, 0)$ , we calculate for every  $L$  the market clearing price of assets assuming that (a) the marginal borrowing firm has a date-0 borrowing requirement of  $L$  and (b) the interest rate on loans is zero. To do this, for every  $L$  we calculate the demand schedule for assets and equate it to asset supply  $\alpha$ .

Consider first any  $L$  that is sufficiently small to satisfy  $kV \leq X_1 - L$ . Note that if the marginal borrowing firm has a date-0 borrowing requirement of  $L$ , the date-1 wealth distribution of industry insiders – i.e. those that invested – will have a support of  $[X_1 - L, X_1]$ .<sup>28</sup> This implies that at any price  $P \leq X_1 - L$  all industry insiders will be able to purchase assets. Since their aggregate date-1 wealth is  $\int_0^L (X_1 - B) dG(B)$ , the demand for assets at any price  $P \in [kV, X_1 - L]$  is given by

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<sup>25</sup>Liquidation is inefficient in that the industry insider purchasing the assets is a second-best user:  $V < X_2$ .

<sup>26</sup>Assuming that  $V < X_1$  generates similar results, but the pricing function developed below is capped at  $V$ .

<sup>27</sup>At the cost of additional complexity  $\alpha$  itself could be endogenized to capture the number of firms experiencing liquidity defaults.

<sup>28</sup>Since the interest rate is zero by assumption, date-1 wealth of a firm that borrowed an amount  $B$  will be  $X_1 - B$ .

$$D(P; L) = \begin{cases} \frac{\int_0^L (X_1 - B) dG(B)}{P} & \text{if } P \in (kV, \leq X_1 - L] \\ D \text{ s.t. } D \geq \frac{\int_0^L (X_1 - B) dG(B)}{kV} & \text{if } P = kV \end{cases} \quad (9)$$

Note that since industry outsiders are ready to purchase any amount of assets at a price of  $P = kV$ , demand at this price is perfectly elastic at any quantity greater than  $\frac{\int_0^L (X_1 - B) dG(B)}{kV}$ , the demand stemming from industry insiders.<sup>29</sup>

In contrast, for any price  $P \in (X_1 - L, X_1]$ , firms require a date-1 wealth of at least  $P$  to purchase assets. This implies that demand over this range will be

$$D(P; L) = \frac{\int_0^{X_1 - P} (X_1 - B) dG(B)}{P} \quad (10)$$

Taken together, (9) and (10) define the demand schedule  $D(P; L)$  for any  $L$  satisfying  $kV \leq X_1 - L$ . On the other hand, for  $L$  satisfying  $kV > X_1 - L$ , it is easy to see that demand is given solely by (10) with the addition that at  $P = kV$  demand is perfectly elastic above  $\frac{\int_0^{X_1 - kV} (X_1 - B) dG(B)}{kV}$ .

For any  $L$ , the equilibrium price of assets solves  $D(P; L) = \alpha$ , equating demand and supply. By definition, this equilibrium price is equal to  $P(L, 0)$ . The following corollary characterizes this pricing function (see Figure 6):

**Corollary 2.** For any  $\alpha < \frac{\int_0^{X_1 - kV} (X_1 - B) dG(B)}{kV}$ , define  $L_1^*(\alpha)$  implicitly by  $\frac{\int_0^{L_1^*(\alpha)} (X_1 - B) dG(B)}{kV} = \alpha$ . Further, define  $L_2^*(\alpha)$  implicitly by  $\frac{\int_0^{L_2^*(\alpha)} (X_1 - B) dG(B)}{X_1 - L_2^*(\alpha)} = \alpha$ . Then the pricing function  $P(L, 0)$  satisfies

$$P(L, 0) = \begin{cases} kV & \text{if } L \leq L_1^*(\alpha) \\ (\frac{1}{\alpha}) \int_0^L (X_1 - B) dG(B) & \text{if } L_1^*(\alpha) < L \leq L_2^*(\alpha) \\ X_1 - L_2^*(\alpha) & \text{if } L > L_2^*(\alpha) \end{cases}$$

Further,  $\text{Max}_L \{P(L, 0)\} = X_1 - L_2^*(\alpha)$  is strictly greater than the outside valuation  $kV$ .

*Proof.* See Appendix.

To understand Corollary 2, note that as  $L$  increases, more firms obtain financing, thereby increasing date-1 liquidity available to purchase assets. However, if  $L$  is comparatively low – and in particular, smaller than  $L_1^*(\alpha)$  – aggregate liquidity of industry insiders is not sufficient to clear the market for assets. The marginal buyers, therefore, are industry outsiders, implying that the market

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<sup>29</sup>For this reason, we also need not consider any price less than  $kV$ .



clearing price is  $kV$ .<sup>30</sup> As  $L$  increases beyond  $L_1^*(\alpha)$ , industry-insider aggregate liquidity becomes sufficiently large to enable them to become the marginal buyers of assets. In this region, increases in  $L$  shift out the demand schedule for assets of industry insiders, implying that the price of collateral will increase as well. Finally, beyond  $L_2^*(\alpha)$ , increases in  $L$  do not affect the equilibrium price of assets which remains constant at  $X_1 - L_2^*(\alpha)$ . At this price, aggregate liquidity is large enough to clear the entire supply of assets. Further increases in the liquidation value  $L$  do increase aggregate liquidity, but only by adding firms with internal wealth lower than the price  $X_1 - L_2^*(\alpha)$ . Thus, even though there are more potential buyers, the equilibrium price remains constant at  $X_1 - L_2^*(\alpha)$ .

It is easy to see that at any exogenous  $L$ , the pricing function  $P(L, 0)$  is decreasing in  $\alpha$ : Since the demand for assets is determined by (the finite) available date-1 liquidity and is not perfectly elastic at the fundamental value  $V$ , the price must drop to clear the market. This yields the following proposition:

**Proposition 5.** *For any outside value  $kV < \frac{X_1}{2}$ , there exists an  $\bar{\alpha} < \frac{\int_0^{X_1-kV} (X_1-B)dG(B)}{kV}$ , such that for any  $\alpha > \bar{\alpha}$ , regardless of the stance of monetary policy, the equilibrium value of collateral will be  $kV$  and aggregate lending will equal  $\bar{B}^{-1}(kV)$ .*

*Proof.* See Appendix.

Proposition 5 shows how expectations of asset sales inhibit the effectiveness of monetary policy. Anticipating a large number of assets on the market, banks understand that the price of collateral will be low. As in a credit trap, while lending would have served to increase date-1 liquidity and with it the price of collateral, the implied value of collateral is too small to actually justify the lending. The economy therefore suffers from a lack of liquidity, collateral prices are depressed at liquidity pricing levels, and lending remains low regardless of the stance of monetary policy.

To emphasize, because  $\bar{\alpha} \leq \frac{\int_0^{X_1-kV} (X_1-B)dG(B)}{kV}$ , as Corollary 2 shows the collateral pricing function  $P(L, 0)$  *does* increase beyond the outside-valuation of  $kV$  (see Figure 6). However, to enable the value of collateral to increase beyond  $kV$ , banks need to supply liquidity to industry insiders, increasing their liquidity sufficiently so that they, rather than the industry outsiders, are the marginal buyers of asset. This, however, will not occur. Regardless of the stance of monetary policy, banks will not lend any level of loan supply greater than  $\bar{B}^{-1}(kV)$  since the implied value of

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<sup>30</sup>Indeed, when  $\alpha > \frac{\int_0^{X_1-kV} (X_1-B)dG(B)}{kV}$ , it is easy to show that  $P(L, 0)$  equals  $kV$  for all  $L$ . Since asset supply is comparatively large, the marginal buyer will always be an industry outsider.

collateral is not sufficiently high to enable the lending. A form of ‘asset overhang’ therefore arises: to push up asset prices, banks need to increase lending sufficiently forcefully. Because of expected asset sales, however, the level of lending required to generate an increase in the value of collateral above  $kV$  is larger than what can be supported by the resultant value of collateral. In equilibrium, expectations of asset sales depress lending and, further, make monetary policy ineffective beyond a certain threshold.

Because the value of collateral is constant at  $kV$  for low levels of liquidity and only begins to rise when insider liquidity is sufficiently large, a convexity arises which may bring about a quantitative easing, jump-start equilibrium. This is formalized in the following proposition (see also Figure 7):

**Proposition 6.** *Assume that  $g(0) = 0$ . If the outside value  $kV$  is sufficiently low, there exist an  $\alpha_1$  and  $\alpha_2$  such that for any asset supply  $\alpha \in [\alpha_1, \alpha_2]$ , a policy of quantitative easing will be successful in increasing lending. Specifically, for any  $\alpha \in [\alpha_1, \alpha_2]$  there exist an  $R_1$  and  $R_2$  such that monetary policy is effective at  $R_1$ , ineffective over the region  $(R_1, R_2)$ , and effective at  $R_2$ .*

Proof: See Appendix

The intuition behind Proposition 6 is similar to that behind Proposition 5. At any  $\alpha \in [\alpha_1, \alpha_2]$ , to increase the value of collateral above  $kV$ , the Central Bank must inject liquidity sufficiently forcefully. Doing so ensures that industry insiders have sufficient liquidity to be the marginal buyers of the assets. On the other hand, because  $\alpha$  is in an intermediate range, the asset overhang effect is dampened.<sup>31</sup> This enables sufficiently forceful liquidity injections to give rise to a high collateral and high lending jump start equilibrium in which quantitative easing is effective.

## Bargaining and Collateral Values

Rather than a competitive market, we consider now the case where the price of repossessed assets is determined through a bargaining process. Assume that if a bank repossesses a firm’s assets in date 1, it must redeploy and sell them to other industry participants. Each firm will be associated with  $N$  other firms, randomly drawn from the population, who are potential operators of that firm’s assets;  $N$  should be thought of as characterizing the degree of redeployability of firm assets.<sup>32</sup> However,

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<sup>31</sup>It is easy to see that  $\alpha_2$  is smaller than the  $\bar{\alpha}$  of Proposition 5.

<sup>32</sup>In the context of physical assets, low redeployability may stem from the high degree of specialization required to operate the assets. Alternatively, in the context of banks providing loans to institutional investors that are collateralized by financial assets, low redeployability corresponds to situations in which the financial assets are sufficiently complex that there are a limited number of investors with the knowledge to value and actively trade in them.

of the  $N$  potential users of a particular firm's assets, only those firms that invest in the project in date-0 acquire the know-how to actually operate the assets.<sup>33</sup> Operating the assets implies using them to generate  $X_2$  in date 2. Finally, for simplicity, we assume that the bank has all the bargaining power in determining the price to be paid for the repossessed assets. (Abandoning this assumption only increases the role that corporate liquidity plays in determining collateral values and lending.)

The expected price that the bank will obtain for a firm's repossessed asset is determined as follows. First, note that all potential users have a period 1 cash balance which is less than  $X_2$ ; a firm that borrowed  $B$  to invest has cash balance  $X_1 - B(1 + r)$ , where  $X_1$  was assumed to be less than  $X_2$ .<sup>34</sup> Each of the potential users will therefore be willing to pay up to the full amount of their cash holdings to acquire the repossessed assets. Since the bank is assumed to have all of the bargaining power, this implies that the amount that the bank will obtain for the repossessed assets will be the entire cash balance of the *wealthiest* firm that can operate the assets. Since only firms that invest in date-0 have the know-how to operate the assets, the expected price that the bank will obtain for repossessed assets is

$$P(B^*, r^*) = \int_0^{B^*} (X_1 - B(1 + r^*)) dG_{(1)}^N, \quad (11)$$

where  $B^*$  is the marginal firm obtaining financing,  $r^*$  is the equilibrium interest rate, and  $G_{(1)}$  is the 1st order statistic of  $N$  draws of  $G$ .

Equation (11) shows that liquidity in the corporate sector in period 1 affects the price banks obtain for repossessed assets in two ways. As  $B^*$  increases, the number of potential buyers with know-how to purchase assets increases. Naturally, this serves to increase the expected price of collateral. Second, all else equal, reductions in the equilibrium interest rate increase liquidity available to firms to purchase assets, again increasing the liquidation value of assets.

Consider now the determinants of the shape of  $P(L, 0)$ , which as described in Proposition 4,

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<sup>33</sup>Prior to raising financing, firms can be thought of as empty shells run by entrepreneurs. Once an entrepreneur raises financing from a bank she employs the capital to start a firm, the subsequent running of which provides the know-how to utilize industry assets. While simplifying the exposition, the assumption that without investing firms do not gain the know-how to operate assets is not necessary.

<sup>34</sup>Recall that the assumption that  $X_1 < X_2$  highlights the role of lack of period 1 liquidity in determining the value of collateral.

influences the effectiveness of monetary policy. For intuition, assume first that  $N = 1$  so that the assets of each firm have only one other potential operator. From (11), we have that

$$P(L, 0) = \int_0^L (X_1 - B)dG, \quad (12)$$

so that the condition for convexity of  $P(L, 0)$  at  $L$  is:

$$(X_1 - L) \frac{g'(L)}{g(L)} > 1. \quad (13)$$

The intuition for (13) is as follows. An incremental increase in the liquidation value  $L$  increases the universe of firms that obtain funding, and which therefore have the know-how to operate repossessed assets. Formally, from (12), the marginal effect of an increase in the liquidation value  $L$  is  $(X_1 - L)g(L)$ . This represents two effects. The incremental potential buyer that is added will have internal wealth of  $X_1 - L$ . However, for any given firm, the probability that this incremental buyer will be the particular buyer that can operate that firm's assets is  $g(L)$ . Therefore, the expected effect of an incremental increase in liquidation value on the implied price the bank will obtain for repossessed assets is the multiplication of the two incremental effects:  $(X_1 - L)g(L)$ .

Note though that as  $L$  increases, the incremental firm able to purchase assets will have diminishing internal cash:  $(X_1 - L)$  decreases in  $L$ . Therefore, the marginal effect of an increase in liquidation value on the price of assets will be increasing in  $L$ , when the increase in the expected number of purchasers is sufficiently large to dominate the reduction in the internal wealth associated with the additional marginal purchasers. Put differently, the pricing function will be convex when the increase in  $g(L)$  dominates the reduction in  $(X_1 - L)$ . Formally, as stated in (13), this occurs when the elasticity of the first effect  $\frac{g'(L)}{g(L)}$  dominates the (inverse) of the elasticity of the second effect  $(X_1 - L)$ .

More generally, consider now the case where there are  $N > 1$  potential buyers for each individual firm's assets. From (11), the condition for convexity then becomes:

$$(X_1 - L) \left( \frac{g'(L)}{g(L)} - \frac{(N-1)}{1 - G(L)} \right) > 1. \quad (14)$$

Since increases in  $N$  reduce the lefthand side of (14), greater asset redeployability increases the prevalence of a concave pricing function. The intuition is as follows. An increase in the liquidation value of assets,  $L$ , enables more firms to obtain financing which increases the universe of potential buyers of collateral. However, when assets are very redeployable, increasing the universe of potential buyers – i.e. making the market thicker – is not as crucial in increasing the expected purchase price; Since assets are easily redeployable, the probability of finding a firm with high net worth with which to pay for the assets, as well as the know-how to actually operate the assets, is comparatively high.<sup>35</sup> Increasing  $L$  will therefore have diminishing impact in this case. In contrast, it is when assets are not easily redeployable that it will be particularly useful to expand the universe of potential buyers – i.e. the pricing function will exhibit a convexity.

Since a convexity in the pricing function is associated with the jump-start equilibrium, a policy of quantitative easing is more likely to be useful when asset redeployability is low. In such a case, increasing the universe of firms with the know-how to buy repossessed assets is particularly important. When injecting liquidity sufficiently forcefully, the increase in the potential number of buyers of collateral will have a disproportionate effect on the expected purchase price of assets, enabling a policy of quantitative easing to be successful. In contrast, high asset redeployability will tend to be associated with a concave pricing function, and hence with a credit trap equilibrium. As shown in Proposition 4, a strategy of quantitative easing will be powerless in such a case.

## Search Costs

It is easy to see that search costs introduce convexities in the pricing function which can bring about jump-start, quantitative easing equilibria. Consider a pricing function  $P$  that describes the liquidation value of assets assuming that a bank sells the assets to an industry insider (for example, as described in the bargaining case above). We add now the assumption that in order to actually find a specialized industry buyer, the bank pays a search cost,  $C$ . This can be thought of as the actual monetary cost, or the opportunity cost of managerial time and effort, associated with finding a buyer. Alternatively, we assume that banks can opt to avoid incurring the search costs by selling liquidated assets to industry outsiders for a low valuation that we normalize to zero.

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<sup>35</sup>For example, in the extreme case, when  $N$  diverges to infinity, the pricing function  $P(L, 0)$  converges to  $X_1$ . This is because with  $N$  very large, there will always be a firm with internal wealth close to  $X_1$  that can operate the assets.

Denoting by  $\tilde{P}$  the pricing function which incorporates the assumption of search costs, we have that:

$$\tilde{P}(L, 0) = \begin{cases} 0 & \text{if } P(L, 0) < C \\ P(L, 0) - C & \text{otherwise} \end{cases}$$

Put simply, if the price obtained by selling the asset to industry insiders is comparatively small, the bank will choose to sell assets to an industry outsider.

Since the pricing function is now constant for low values of  $L$  and only then begins to increase, a convexity arises (see Figure 8). This, in turn, will bring about a possibility of a quantitative easing, jump-start equilibrium. This is formalized in the following proposition:

**Proposition 7.** *Assume that the pricing function  $P(L, 0)$  is concave with  $\frac{\partial P(L, 0)}{\partial L}|_{L=0} > 1$ . Define  $L^*$  to be the point that satisfies  $\frac{\partial P(L^*, 0)}{\partial L} = 1$ . For any cost  $P(0, 0) < C \leq P(L^*, 0) - L^*$  there exists an  $\bar{R}$  such that monetary policy will be ineffective over the region  $[0, \bar{R})$  but effective at  $\bar{R}$ . A policy of quantitative easing over the region  $[0, \bar{R}]$  will thus be successful in increasing lending.*

Proof: See Appendix

The intuition of Proposition 7 is quite simple. As discussed above, if the price of assets is comparatively small when sold to industry insiders – say, because the market is too thin or corporate liquidity amongst industry insiders is too low – banks will choose to ignore this market and avoid the associated search costs by selling directly to industry outsiders. Therefore, to increase the value of collateral above the industry outsider valuation, the central bank must inject liquidity sufficiently forcefully to ensure that the industry insider market is developed – i.e. that there are a sufficient number of industry insiders with high enough internal liquidity. As in a quantitative easing equilibrium, the rise in collateral values above the outsider valuation will support, in equilibrium, additional lending. The condition that  $P' > 1$  ensures that the liquidity pricing effect is sufficiently strong so as to generate an equilibrium in which the value of collateral is above the outside value despite the search cost.<sup>36</sup>

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<sup>36</sup>On a related point, note also that search costs cannot be too high since otherwise banks will choose to sell to outside buyers regardless of the depth of the insider market.

## 6. Fiscal Policy and Direct Injections of Corporate Liquidity

We have shown how financial frictions and the interplay between liquidity and collateral values hinder the translation of liquidity injections to the financial sector into increased credit and investment. This line of reasoning suggests that *direct* injections of liquidity into the corporate sector may be beneficial. In particular, by circumventing financial intermediaries, liquidity provision to the corporate sector will increase collateral values directly, enabling firms to extract liquidity from banks on their own.

Fiscal policy is one tool by which the government can directly affect liquidity in the corporate sector without relying on financial intermediation. For example, by reducing the corporate tax rate or introducing an investment tax credit, the government can increase the level of internal liquidity available to firms – liquidity which firms can then use to purchase assets and hence raise collateral values. Alternatively, increasing direct government spending may serve to increase internal corporate liquidity, for example when individuals use funds to increase consumption. Finally, direct government loans to firms will also serve to increase corporate liquidity, particularly when these are provided at a rate lower, and in amounts higher, than that provided by the financial sector.<sup>37</sup> In this section we show that in a credit trap setting direct injections of liquidity into the corporate sector give rise to a natural synergy between monetary and fiscal policy as well as a fiscal multiplier effect.

Formally, we examine the effect of government injections of liquidity into the corporate sector by analyzing exogenous shocks to date-1 cash flow,  $X_1$ .<sup>38</sup> Since we are now interested in the effects of changes in  $X_1$ , we rewrite the direct pricing function,  $P$ , as  $P(B, r; X_1)$ . Assuming that collateral prices can only increase when date-1 corporate liquidity increases in all firms, we have that  $P$  is increasing in  $X_1$ .<sup>39</sup>

Consider the effect of an increase in date-1 cash flow,  $X_1$ , in the case of a credit trap equilibrium. We have the following proposition:

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<sup>37</sup>During the financial crisis of 2008-2009 the Federal Reserve provided direct loans to the corporate sector using the Commercial Paper Funding Facility. While not constituting monetary policy (nor fiscal policy) in the standard sense, the program allowed the government to bypass financial intermediaries along the lines discussed here.

<sup>38</sup>For example, decreases in the corporate tax rate  $\tau$  will increase firm after-tax date-1 income,  $(1 - \tau)X_1$ .

<sup>39</sup>For simplicity, we make the assumption that  $P$  is *strictly* increasing in  $X_1$ . Alternatively, we could assume that  $P$  is increasing in  $X_1$  in a region  $[0, \bar{X}_1]$ . With the latter assumption, all of the following propositions should be interpreted as confined to that region.

**Proposition 8.** *Assume that the conditions in Proposition 3(ii) hold so that the economy is in a credit trap. An increase in  $X_1$  increases the maximal aggregate loan supply which the banking sector can successfully lend to firms.*

The intuition for Proposition 8 is quite simple. An exogenous increase in date-1 liquidity increases collateral values which, in turn, supports higher lending by banks. The increased lending by banks further increases corporate liquidity and so collateral values increase further. In this way, liquidity-induced increases in collateral values and lending amplify one another. By circumventing banks and directly injecting liquidity into the corporate sector, fiscal policy eases a credit trap equilibrium.

This process is demonstrated in Figure 9. At  $X_1^{low}$  the economy is in a credit trap equilibrium, with the maximal liquidation value of assets being the fixed point  $L_1$  satisfying  $P(L_1, 0; X_1^{low}) = L_1$ . The corresponding maximal aggregate amount of loans that banks will be willing to provide to firms is then  $\bar{B}^{-1}(L_1)$ . Consider now the effect of a shift from  $X_1^{low}$  to  $X_1^{high}$  arising from an expansion in fiscal policy. The associated increase in corporate liquidity will raise the collateral pricing function  $P$ . The implied value of assets assuming a liquidation value of  $L_1$  and an interest rate of zero rises from  $P(L_1, 0; X_1^{low})$  to  $L_2 \equiv P(L_1, 0; X_1^{high})$ . This higher level of collateral values will support a higher level of lending, equal to  $\bar{B}^{-1}(L_2)$ , which will imply a still higher liquidation value of  $L_3 \equiv P(L_2, 0; X_1^{high})$ . The initial increase in collateral values due to the liquidity provided by the fiscal policy expansion is amplified in the feedback loop between lending, liquidity, and collateral values. The process converges at the point in the figure marked  $L_H$  satisfying  $P(L_H, 0; X_1^{high}) = L_H$ .

Indeed, the feedback loop between liquidity injections, collateral values, and lending generates a fiscal policy multiplier effect. This is described in the following proposition:

**Proposition 9.** *Assume that the economy is in a credit trap as described in Proposition 3(ii) where the supply of loanable funds is greater than the maximal lending capacity of banks,  $R_{max}$ , and the equilibrium liquidation value of assets is at its maximal level,  $L^*$ . Assume that the government transfers  $\delta$  in period 1 to every firm which invested in period 0. Defining  $T$  as the aggregate wealth transfer to firms and  $W(T)$  as the aggregate period-1 wealth of firms, we have that  $\frac{\partial W(T)}{\partial T} > 1$ . Further, the fiscal policy multiplier increases when the liquidity pricing effect is greater:  $\frac{\partial W(T)}{\partial T}$  increases with  $\frac{\partial P}{\partial L}(L^*, 0)$ .*



As discussed above, the government transfer to firms raises expected corporate liquidity at date-1, and with it the expected equilibrium collateral value. This, in turn, allows greater lending and investment, which further increases period-1 firm wealth. The effect of the transfer is therefore multiplied through the interplay of lending, corporate liquidity, and investment.<sup>40</sup> As would be expected, the strength of the multiplier effect is larger when the impact of additional liquidity on the value of collateral is greater.

In sum, by circumventing financial intermediaries and providing direct liquidity injections, fiscal policy increases the maximal borrowing capacity of firms and enables liquidity to be extracted from banks. With fiscal policy in place, monetary policy can *push* out liquidity from the banking sector to the corporate sector, employing in the process the positive feedback loop between lending, liquidity and increased collateral values. Lending is therefore jump-started through a push-pull mechanism, where monetary policy serves to push and fiscal policy serves to pull out liquidity from financial intermediaries into firms. As such, the model therefore implies a natural synergy between monetary and fiscal policy.

### 6.1. The Effects of Initial Liquidity

In this final section we analyze how the initial level of liquidity in the corporate sector affects the success of monetary policy interventions. We show that if date-0 corporate liquidity is sufficiently low, such interventions will not be successful, in that banks will not increase lending and liquidation values will be stuck at low liquidity-pricing levels. Credit traps thus arise when initial corporate liquidity at the time of intervention is low.

Formally, we analyze the model's comparative statics with respect to the distribution of borrowing requirements,  $G$ . We make the added assumption that the pricing function  $P$  is continuous in  $G$  under the  $\mathcal{L}^1$  metric, implying simply that if  $G$  and  $G'$  are similar distributions of borrowing requirements, the implied values of collateral for each distribution will be similar as well. Since each firm's borrowing requirement  $B$  is equal to  $I - A$  (i.e the difference between its internal wealth and the required investment outlay) increases in firm borrowing requirements are associated with decreases in initial corporate liquidity. Thus, if  $G$  and  $G'$  are two distributions of firm borrowing

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<sup>40</sup>As explained, one way to think of the transfer  $\delta$  is through a reduction in the corporate tax rate. Reducing the rate from  $\tau_1$  to  $\tau_2$  involves a transfer of  $\delta = (\tau_1 - \tau_2)X_1$ . Focusing on tax rate changes, it is easy to see that lowering the tax rate may actually serve to increase tax revenue; Reducing the tax rate will increase the tax base, since more firms can obtain financing and invest due to the equilibrium effect on collateral values.

requirements with  $G'$  first order stochastically dominating  $G$ , liquidity in the corporate sector is higher under the distribution  $G$  as compared to the distribution  $G'$ . Further, we denote for every distribution  $G$ , the aggregate date-0 liquidity in the corporate sector as  $W(G) := \int_0^I (I - B)dG(B)$ .

Clearly, the smallest level of aggregate corporate liquidity in date-0 is zero – this is the situation in which all firms have zero internal wealth, and thus all firms have a borrowing requirement of  $I$ . For what follows, it turns out useful to define  $P_0$  to be the date-1 price of collateral assuming that (1) aggregate liquidity was zero in date-0 and (2) in date-0 all firms were provided financing at a zero interest rate, so that in date-1 all firms have liquidity  $X_1 - I$ .

The next proposition shows that if  $X_1$  is sufficiently small, low levels of date-0 aggregate corporate liquidity necessarily involve credit traps:

**Proposition 10.** *Assume that  $X_1$  is sufficiently small so that  $P_0 < I$ . Then there exists a threshold level of aggregate liquidity  $\bar{W}$  and a level of loan supply  $R^* < R_{max}$  such that for any distribution of date-0 liquidity  $G$  with aggregate liquidity  $W < \bar{W}$ , monetary policy will be ineffective beyond  $R^*$ .*

The intuition of Proposition 10 is as follows. In providing liquidity to the corporate sector, banks must rely on the aggregate liquidity *already present* in the corporate sector, as this determines, in part, the value of collateral and the strength of firms' balance sheets. Therefore, when injecting liquidity into the banking sector in the hope of increasing bank lending and jump starting the feedback loop between lending and collateral, the central bank must also rely on the initial level of liquidity in the corporate sector. In effect, the central bank is leveraging the initial level of aggregate liquidity to inject additional liquidity into the corporate sector. Proposition 10 then states that if this initial level of aggregate liquidity is sufficiently low, the central bank's ability to leverage existing liquidity to increase lending will be limited – i.e. the economy will be in a credit trap equilibrium where banks will not increase lending beyond a certain level, regardless of shifts in the loan supply.

The next proposition states that if an economy is in a credit trap, decreases in date-0 corporate liquidity intensify the severity of the credit trap: The maximal level of lending and the maximal value of collateral both decrease.

**Proposition 11.** *Consider an economy with a date-0 distribution of borrowing needs  $G_1$  which is in a credit trap equilibrium. Denote by  $R_1^*$  the maximal aggregate lending and  $L_1^*$  the maximal*

collateral value in this credit trap equilibrium. If  $G_2$  stochastically dominates  $G_1$  (implying that date-0 liquidity in the corporate sector is higher under  $G_1$ ) then under the distribution  $G_2$ :

- (i) The maximal aggregate level of loans provided by banks,  $R_2^*$ , will be smaller than  $R_1^*$ .
- (ii) The maximal value of collateral,  $L_2^*$ , will be smaller than  $L_1^*$ .

Proposition 10 and 11 make clear the importance of initial aggregate liquidity when the central bank tries to intervene and inject liquidity into the banking sector. Proposition 10 states that if aggregate liquidity is sufficiently low at the point of intervention, the economy will be stuck in a credit trap, while Proposition 11 states that as aggregate liquidity decreases, this credit trap becomes more severe. Put together, the propositions show that monetary intervention may arrive too late: If liquidity is sufficiently low in the corporate sector, monetary expansions will not easily convey additional liquidity from financial intermediaries to firms.

## 7. Conclusion

We study the limitations of the ‘credit channel’ in transmitting monetary policy into real economic outcomes. Developing a model that relies on the interplay between lending, liquidity, and collateral values, we identify three types of equilibria that may arise. The first equilibrium is one in which monetary policy successfully influences aggregate lending activity. In this conventional equilibrium, expansionary monetary policy translates into an increase in collateral values and lending. In the second equilibrium, which we call a credit trap equilibrium, the transmission mechanism of the credit channel fails. Any easing of monetary policy beyond a certain level is completely ineffective in increasing aggregate lending or collateral values. In the third equilibrium, called the jump-start equilibrium, a policy of quantitative easing will be successful in increasing bank lending: monetary policy can be effective, but only when the central bank injects a sufficiently large amount of reserves into the banking sector.

While monetary policy on its own is ineffective in a credit trap equilibrium, we show that *fiscal* policy acting in conjunction with monetary policy can be useful in easing credit traps and increasing bank lending. By circumventing financial intermediaries and directly injecting liquidity into the corporate sector, expansionary fiscal policy can increase collateral values. With collateral values increased, banks can then provide loans to firms. Our paper therefore proposes a natural complementarity between fiscal and monetary policy in stimulating lending.

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## Appendix

**Proof of Proposition 1.** First assume that  $L^*$  satisfies  $p(L^*; R) = L^*$ . By definition,  $B^*(L; R)$  and  $r^*(L; R)$  are the associated equilibrium marginal borrowing firm and market clearing interest rate associated with the exogenous pair of liquidation values and loan supply  $(L, R)$ . By construction, therefore, the vector  $(R, r^*, L^*, B^*)$  satisfies conditions (i) through (iii) of a market equilibrium. Further, we have that  $L^* = p(L^*; R) = P(B^*(L^*; R), r^*(L^*; R))$ , where the second equality results from the definition of the pricing function  $p$ . Thus, condition (iv) of the market equilibrium is satisfied as well, guaranteeing that  $R, r^*, L^*, B^*$  is indeed an equilibrium.

Suppose now that the vector  $(R, r^*, L^*, B^*)$  is a market equilibrium. By Section 3, it is easy to see that for every exogenous pair  $(L^*, B^*)$  there is a unique equilibrium marginal borrowing firm and market clearing interest rate. Thus,  $B^*(L^*; R) = B^*$  and  $r^*(L^*; R) = r^*$ . Since  $(R, r^*, L^*, B^*)$  is a market equilibrium, condition (iv) implies that  $L^* = P(B^*, r^*) = P(B^*(L^*; R), r^*(L^*; R)) = p(L^*; R)$ .  $L^*$  is therefore a fixed point of  $p$  as required.

As a final point, note that existence of at least one equilibrium is guaranteed by the fact that  $p$  is continuous (since  $P$  is continuous) and bounded from above by  $X_1$ .

**Proof of Proposition 2.** Assume first that  $L < \bar{B}(R)$ . We have then that  $\int_0^L B dG(B) < \int_0^{\bar{B}(R)} B dG(B)$ . As shown in Section 3, the left hand side of this inequality is equal to  $D(0)$ , the demand for loans at a zero interest rate, while the right hand side of the inequality equals  $R$  (by definition of  $\bar{B}(R)$ ). Thus, we have that  $D(0) < R$ : Loan demand is smaller than loan supply even at the lower bound of  $r = 0$ , implying that the equilibrium interest rate will indeed be zero. (Recall that to obtain a positive interest rate, the market for loanable funds must clear with equality.)

Note now that  $\int_0^{\bar{B}(R)} B dG(B) \leq R \leq \int_0^I B dG(B)$  implies that  $L < \bar{B}(R) \leq I$ . Since  $L < I$ , therefore, at a zero interest rate the marginal borrowing firm will have a borrowing requirement of  $L$ . This is because the marginal firm *able* to borrow has a borrowing requirement of  $L$ , while the marginal firm that would *like* to borrow has the maximal borrowing requirement of  $B = I > L$ . We therefore have that the equilibrium interest rate associated with the pair  $(L, R)$  will be  $r^* = 0$  and  $B^* = L$ . By definition, the indirect pricing function is therefore  $p(L; R) = P(L, 0)$ . Finally, in this equilibrium, the level of loan supply actually lent out will equal the effective loan demand at the zero interest rate,  $D(0) = \int_0^L B dG(B)$ . As shown above, this is strictly smaller than  $R$ .

Consider now the case where  $L \geq \bar{B}(R)$ . As above, this implies that  $D(0) \geq R$ . Since demand for loanable funds monotonically decreases to zero as  $r$  diverges to infinity, there must be an  $r^*$  at which demand and supply equate:  $D(r^*) = R$ . By definition, this  $r^*$  will be the equilibrium interest rate, and further, at this rate the full amount  $R$  will be lent out. Since the full amount  $R$  is lent out, by definition of the function  $\bar{B}$ , this implies that the marginal firm that obtains financing has a borrowing requirement of  $B^* = \bar{B}(R)$ . By equation (3) the marginal firm obtaining finance at an interest rate  $r$  has a borrowing requirement of  $\min[\frac{L}{(1+r)}, \frac{(X_1+X_2-I)}{r}]$ , which implies that  $r^*$  satisfies  $\bar{B}(R) = \min[\frac{L}{(1+r^*)}, \frac{(X_1+X_2-I)}{r^*}]$ . Finally, since the equilibrium marginal borrowing firm and interest rate are  $\bar{B}(R)$  and  $r^*$  respectively, we have that the indirect pricing function satisfies

$$p(L; R) = P(\bar{B}(R), r^*).$$

**Proof of Corollary 1.** Consider first a liquidation value  $L$  with  $L \leq \bar{B}(R)$ . By Proposition 2, we have that in this region  $p(L; R) = P(L, 0)$ . Since  $P$  is increasing in  $L$ ,  $p$  will be increasing in  $L$  as well in this region (recall that, unless stated otherwise, throughout the paper ‘increasing’ refers to weak monotonicity.)

Similarly, by Proposition 2, if  $L > \bar{B}(R)$  we have that

$$p(L; R) = P(\bar{B}(R), r^*), \tag{15}$$

with  $r^*$  satisfying  $\bar{B}(R) = \min[\frac{L}{(1+r^*)}, \frac{(X_1+X_2-I)}{r^*}]$ . Therefore,  $r^*$  is increasing in  $L$  (recall that  $R$  is exogenous and therefore constant in this comparative static). Thus, since  $P$  is decreasing in  $r$  by (15) we have that  $p$  is decreasing in  $L$ .

**Proof of Proposition 3.** Assume first that  $P(\bar{B}(R), 0) < \bar{B}(R)$  and assume by contradiction that there exists an equilibrium  $(R, r^*, L^*, B^*)$  in which  $R$  is lent out. By Corollary 1,  $p(L; R)$  is decreasing in  $L$  over the region  $L > \bar{B}(R)$ . Further, By Proposition 2,  $p(\bar{B}(R); R) = P(\bar{B}(R), 0)$ . Since by assumption  $P(\bar{B}(R), 0) < \bar{B}(R)$ , we therefore have that  $p(\bar{B}(R); R) < \bar{B}(R)$  and that  $p(L; R)$  decreases over  $L > \bar{B}(R)$ . Since any equilibrium liquidation value must satisfy  $p(L^*; R) = L^*$ , this implies that  $L^* < \bar{B}(R)$ . By Proposition 2(1)(i), however, this implies that an amount strictly less than  $R$  is lent out, contrary to the original assumption.

Suppose now that  $P(\bar{B}(R), 0) \geq \bar{B}(R)$  and assume that loan supply is  $R$ . To show that there exists an equilibrium in which  $R$  is lent out, note again that by Proposition 2, we have that  $p(\bar{B}(R); R) = P(\bar{B}(R), 0)$ . This implies that  $p(\bar{B}(R); R) \geq \bar{B}(R)$ . Since  $p$  is continuous and bounded from above by  $X_1$  there exists an  $L^* \geq \bar{B}(R)$  which satisfies  $p(L^*; R) = L^*$ . By Proposition 1 this  $L^*$  is an equilibrium liquidation value. Further, by section (2)(i) of Proposition 2, since  $L^* \geq \bar{B}(R)$  the entire loan supply  $R$  will be lent out.

We therefore have that a loan supply  $R$  will be lent out in its entirety if and only if  $P(\bar{B}(R), 0) \geq \bar{B}(R)$ . Thus, for any  $R = \bar{B}^{-1}(L)$ , we have that  $R$  is lent out if and only if  $P(L, 0) \geq L$ .

**Proof of Proposition 4.** The proposition is a direct result of Proposition 2 and Proposition 3. Consider first the case of the conventional equilibrium. Since  $P(L, 0) > L$  for all  $0 < L \leq I$ , we know by Proposition 3 that monetary policy is effective at any  $R \in [\bar{B}^{-1}(0), \bar{B}^{-1}(I)] = [0, R_{max}]$  where  $R_{max} = \int_0^I B dG(B)$ .

Consider now the effect of shifts in loan supply  $R$  on the equilibrium liquidation value of assets. For any  $R \leq R_{max}$  we have that  $\bar{B}(R) \leq I$ . Therefore, the equilibrium liquidation value can't satisfy  $L \leq \bar{B}(R)$  since in this region by Proposition 2 we have that  $p(L; R) = P(L, 0) > L$ . Thus, the equilibrium liquidation value must be in the region  $L > \bar{B}(R)$ . Since  $p$  is decreasing in  $L$ , and the equilibrium liquidation value must satisfy  $p(L; R) = L$  it is clear that the equilibrium liquidation value is unique. Further, by Proposition 2, in this region  $p(L; R) = P(\bar{B}(R), r^*)$  where

$r^*$  satisfies  $\bar{B}(R) = \min[\frac{L}{(1+r^*)}, \frac{(X_1+X_2-I)}{r^*}]$ . It is easy to see from this that  $\frac{\partial r^*}{\partial R} \leq 0$  and also  $\frac{\partial r^*}{\partial L} \geq 0$ .

Consider, therefore, for each loan supply  $R \leq R_{max}$  the equilibrium liquidation value  $L(R)$  that satisfies  $P(\bar{B}(R), r^*(L; R)) = L(R)$ . Differentiating both sides by  $R$  and rearranging yields:

$$L'(R) = \frac{\frac{\partial P}{\partial B} \frac{\partial B}{\partial R} + \frac{\partial P}{\partial r} \frac{\partial r^*}{\partial R}}{1 - \frac{\partial P}{\partial r} \frac{\partial r^*}{\partial L}}. \quad (16)$$

Since  $\frac{\partial P}{\partial B} \geq 0$ ,  $\frac{\partial B}{\partial R} \geq 0$ ,  $\frac{\partial P}{\partial r} \leq 0$ ,  $\frac{\partial r^*}{\partial R} \leq 0$ , and  $\frac{\partial r^*}{\partial L} \geq 0$ , we have that  $L'(R) \geq 0$  as was to be shown.

Next, consider the credit trap equilibrium described in case (ii) of the proposition. Since  $P(L, 0) \geq L$  for  $0 < L \leq L^*$ , by Proposition 3 monetary policy is effective over the region  $[\bar{B}^{-1}(0), \bar{B}^{-1}(L^*)]$ . Further, since  $P(L, 0) < L$  for  $L > L^*$  by Proposition 3 monetary policy is ineffective for any  $R > \bar{B}^{-1}(L^*)$ . Maximal equilibrium lending is therefore  $R^* = \bar{B}^{-1}(L^*)$ .

Consider now a loan supply  $R$  greater than  $R^*$ . We have that  $\bar{B}(R) > \bar{B}(R^*) = L^*$ . Therefore, the equilibrium liquidation value cannot be greater than  $\bar{B}(R)$  since for any  $L > L^*$  according to the assumption in case (ii) of the Proposition  $P(L, 0) < L$ . Since, by Proposition 2,  $p(L; R) \leq P(L, 0)$  this implies that in this region  $p(L; R) < L$ . To be an equilibrium, however,  $L$  must satisfy  $p(L; R) = L$ . Thus, the equilibrium liquidation value must have  $L \leq \bar{B}(R)$ . In this region, though, by Proposition 2,  $p(L; R) = P(L, 0)$ . Thus, the only equilibrium liquidation value is  $L^*$  since according to case (ii) of the proposition,  $P(L^*, 0) = L^*$ ,  $P(L, 0) > L$  for  $0 < L < L^*$ , and  $P(L, 0) < L$  for  $L > L^*$ .

Case (iii) of the jump start equilibrium is proved in a similar manner. Since  $P(L, 0) < L$  for  $L_1 < L < L_2$ , by Proposition 3 we have that monetary policy is ineffective at any  $R \in (\bar{B}^{-1}(L_1), \bar{B}^{-1}(L_2)) = (R_1, R_2)$ . Now, since  $P(L_2, 0) = L_2$ , by Proposition 3 we have that if loan supply is  $R_2 = \bar{B}^{-1}(L_2)$  it will successfully be lent out in equilibrium.

Consider now any loan supply  $R \in [R_1, R_2]$ . Since  $L_1 \leq \bar{B}(R) < L_2$ , it must be that the associated equilibrium is smaller than  $\bar{B}(R)$ . This is because  $p(L; R)$  is decreasing for  $L > \bar{B}(R)$  and  $p(\bar{B}(R); L) = P(\bar{B}(R), 0) < \bar{B}(R)$  based on the assumption in case (iii) of Proposition 4. Thus  $p(L; R) < L$  for  $L \geq \bar{B}(R)$ , whereas to be an equilibrium equality is needed. Thus, the equilibrium liquidation value satisfies  $L \leq \bar{B}(R)$ . Over this range, though, by Proposition 2,  $p(L; R) = P(L, 0)$ . Thus, based on the assumption in case (iii) in the proposition the only liquidation value is  $L_1$  since  $P(L, 0) > L$  for  $0 < L < L_1$ ,  $P(L, 0) < L$  for  $L_1 < L < L_2$ , and  $P(L_1, 0) = L_1$ . Finally, since  $p(\bar{B}(R_2); R_2) = P(\bar{B}(R_2), 0) = \bar{B}(R_2)$ , we have that  $L_2 = \bar{B}(R_2)$  is an equilibrium liquidation value when loan supply is  $R_2$ .

**Proof of Corollary 2.** Since by assumption  $\alpha < \frac{\int_0^{X_1-kV} (X_1-B)dG(B)}{kV}$ , by continuity there exists an  $L_1^* < X_1 - kV$  with  $\alpha = \frac{\int_0^{L_1^*} (X_1-B)dG(B)}{kV}$ . Consider now a value  $L \leq L_1^*$ . Since  $L_1^* < X_1 - kV$ , we have that  $kV \leq X_1 - L$ . Further, if the marginal firm obtaining financing has borrowing requirement  $L$ , then at an interest rate of zero, date-1 distribution of wealth is  $[X_1 - L, X_1]$ . Since  $kV < X_1 - L$ , we therefore have that at a price of  $P = kV$ , demand by industry-insiders is given by  $\frac{\int_0^L (X_1-B)dG(B)}{kV}$ .



Since industry outsiders are willing to pay  $kV$  for the assets, we have that demand at the price  $P = kV$  satisfies  $D \geq \frac{\int_0^L (X_1 - B) dG(B)}{kV}$ . Because  $\alpha = \frac{\int_0^{L_1^*} (X_1 - B) dG(B)}{kV} \geq \frac{\int_0^L (X_1 - B) dG(B)}{kV}$ , we have that  $P = kV$  is an equilibrium price of assets. To see that any  $P > kV$  cannot be an equilibrium, note that for such  $P$  outsiders are priced out of the market while insider demand is bounded from above by  $\frac{\int_0^L (X_1 - B) dG(B)}{P}$ . Since  $\frac{\int_0^L (X_1 - B) dG(B)}{P} < \frac{\int_0^L (X_1 - B) dG(B)}{kV} \leq \frac{\int_0^{L_1^*} (X_1 - B) dG(B)}{kV} = \alpha$  it cannot be that a price  $P > kV$  is an equilibrium of the asset market. Thus,  $P = kV$  is the unique equilibrium price. By definition, therefore, for any  $L \leq L_1^*$ , the direct pricing function satisfies  $P(L, 0) = kV$ . Define now  $L_2^*(\alpha)$  implicitly by  $\frac{\int_0^{L_2^*(\alpha)} (X_1 - B) dG(B)}{X_1 - L_2^*(\alpha)} = \alpha$ . Because  $L_1^* < X_1 - kV$  and by definition  $\alpha = \frac{\int_0^{L_1^*} (X_1 - B) dG(B)}{kV}$ , we have that  $L_1^* < L_2^*$ .

Consider therefore the demand function  $D(P; L)$  for an exogenous  $L \in (L_1^*, L_2^*]$ .

Because  $\frac{\int_0^{L_2^*(\alpha)} (X_1 - B) dG(B)}{X_1 - L_2^*(\alpha)} = \alpha < \frac{\int_0^{X_1 - kV} (X_1 - B) dG(B)}{kV}$  we have that  $L_2^* < X_1 - kV$ . Since  $L \leq L_2^* < X_1 - kV$ , then  $kV \leq X_1 - L$ . The demand schedule is therefore governed by (9) and (10).

Now, because  $L > L_1^*$  and  $\alpha = \frac{\int_0^{L_1^*} (X_1 - B) dG(B)}{kV}$ , it must be that  $\alpha < \frac{\int_0^L (X_1 - B) dG(B)}{kV}$ . This implies, though, that  $P = kV$  is not an equilibrium price of assets since by (9) demand at  $kV$  is greater than  $\frac{\int_0^L (X_1 - B) dG(B)}{kV}$ .

Since  $L \leq L_2^*$  we have that  $\frac{\int_0^L (X_1 - B) dG(B)}{X_1 - L} \leq \frac{\int_0^{L_2^*(\alpha)} (X_1 - B) dG(B)}{X_1 - L_2^*(\alpha)} = \alpha$ . This, together with the fact that by (10) over the region  $P > X_1 - L$ ,  $D(P; L) < \frac{\int_0^L (X_1 - B) dG(B)}{X_1 - L}$ , it must be that the market clearing price satisfying  $D(P^*; L) = \alpha$  satisfies  $P^* \leq X_1 - L$ . By (9), over this region we have that  $D(P; L) = \frac{\int_0^L (X_1 - B) dG(B)}{P}$ , implying that the market clearing price is  $(\frac{1}{\alpha}) \int_0^L (X_1 - B) dG(B)$ . By definition, this market clearing price is  $P(L, 0)$ .

Consider now an  $L > L_2^*(\alpha)$ . First, assume that  $kV \leq X_1 - L$  so that the demand schedule is governed by (9) and (10). Because  $\frac{\int_0^L (X_1 - B) dG(B)}{X_1 - L} > \frac{\int_0^{L_2^*(\alpha)} (X_1 - B) dG(B)}{X_1 - L_2^*(\alpha)} = \alpha$  and since by (9) over the region  $P < X_1 - L$  we have that  $D(P; L) > \frac{\int_0^L (X_1 - B) dG(B)}{X_1 - L}$  it must be that the market clearing price satisfying  $D(P^*; L) = \alpha$  satisfies  $P^* \geq X_1 - L$ . Over this region, though, by 10 we have that  $D(P; L) = \frac{\int_0^{X_1 - P} (X_1 - B) dG(B)}{P}$ . Thus, the equilibrium price satisfies  $\frac{\int_0^{X_1 - P^*} (X_1 - B) dG(B)}{P^*} = \alpha$ . By definition of  $L_2^*(\alpha)$  this implies that  $P^* = X_1 - L_2^*(\alpha)$ . Assume now that  $kV > X_1 - L$ . This means that the demand schedule is governed by (10). By the same argument, therefore, the equilibrium price of assets is  $P^* = X_1 - L_2^*(\alpha)$ . Thus, regardless of the relation between  $kV$  and  $X_1 - L$ , for all  $L > L_2^*(\alpha)$ , the equilibrium price, is  $P^* = X_1 - L_2^*(\alpha)$ . By definition, therefore  $P(L, 0) = X_1 - L_2^*(\alpha)$  for  $L > L_2^*(\alpha)$ . Finally, since  $L_2^* < X_1 - kV$  we have that  $P(L, 0) = X_1 - L_2^*(\alpha) > kV$ . Note also that because  $G$  is twice differentiable and the smooth pasting conditions at  $L_1^*$  and  $L_2^*$  are satisfied, the function  $P(L, 0)$  is continuous in both  $L$  and the exogenous  $\alpha$ .

**Proof of Proposition 5.** By Corollary 2, for any  $\alpha$  for which  $L_1^*(\alpha) > kV$  and  $L_1^*(\alpha) > X_1 - L_2^*(\alpha)$ , we have that (a)  $L = kV$  is a fixed point of  $P(L, 0)$ ; (b)  $P(L, 0) > L$  for  $L < kV$ ; and (c)  $P(L, 0) < L$  for  $L < kV$ . To see this note that by Corollary 2,  $P(L, 0) = kV$  for all  $L \leq L_1^*(\alpha)$ . Since  $L_1^*(\alpha) > kV$  we have that  $P(kV, 0) = kV$ . By the same argument, for any  $L < kV$  we have that  $P(L, 0) = kV > L$ , while for  $L \in (kV, L_1^*(\alpha)]$  we have that  $P(L, 0) = kV < L$ . Finally, because

$P(L, 0)$  increases for all  $L > L_1^*(\alpha)$  and obtains at  $L = L_2^*(\alpha)$  its maximum value of  $X_1 - L_2^*(\alpha)$ , over the region  $L > L_1^*(\alpha)$ ,  $P(L, 0) \leq X_1 - L_2^*(\alpha) < L_1^*(\alpha) < L$ . We have thus proved all three requirements stated in conditions (a), (b), (c). Given this, the proposition is then a direct result of Corollary 2.

To conclude the proof, it needs to be shown that there exists an  $\bar{\alpha} < \frac{\int_0^{X_1-kV} (X_1-B)dG(B)}{kV}$  such that for all  $\alpha > \bar{\alpha}$ ,  $L_1^*(\alpha) > kV$  and  $L_1^*(\alpha) > X_1 - L_2^*(\alpha)$ . To see this note that since by definition  $\alpha = \frac{\int_0^{L_1^*} (X_1-B)dG(B)}{kV}$ , we have that  $L_1^*(\frac{\int_0^{X_1-kV} (X_1-B)dG(B)}{kV}) = X_1 - kV$ . Since by assumption  $kV < \frac{X_1}{2}$ , we have that  $X_1 - kV > kV$ . Since  $L_1^*(\alpha)$  is increasing in  $\alpha$ , by continuity there exists an  $\bar{\alpha}_1 < \frac{\int_0^{X_1-kV} (X_1-B)dG(B)}{kV}$  such that for all  $\alpha > \bar{\alpha}_1$ ,  $L_1^*(\alpha) > kV$ .

Finally, note that since  $L_2^*(\alpha)$  is implicitly defined by  $\frac{\int_0^{L_2^*(\alpha)} (X_1-B)dG(B)}{X_1-L_2^*(\alpha)} = \alpha$ , we have that  $L_2^*(\frac{\int_0^{X_1-kV} (X_1-B)dG(B)}{kV}) = X_1 - kV$ . Therefore, at  $\alpha^* = \frac{\int_0^{X_1-kV} (X_1-B)dG(B)}{kV}$ , we have that  $L_1^*(\alpha^*) + L_2^*(\alpha^*) = 2(X_1 - kV) > X_1$  where the inequality stems from the fact that  $kV < \frac{X_1}{2}$ . Since  $L_2^*(\alpha)$  is increasing in  $\alpha$ , by continuity there exists an  $\bar{\alpha}_2 < \frac{\int_0^{X_1-kV} (X_1-B)dG(B)}{kV}$ , such that  $L_1^*(\alpha) + L_2^*(\alpha) > X_1$  for all  $\alpha > \bar{\alpha}_2$ . Put differently, for all  $\alpha > \bar{\alpha}_2$ ,  $L_1^*(\alpha^*) > X_1 - L_2^*(\alpha)$ . By defining  $\bar{\alpha} = \min(\bar{\alpha}_1, \bar{\alpha}_2)$  the proposition is proved.

**Proof of Proposition 6.** Assume that  $\alpha < \frac{\int_0^{X_1-kV} (X_1-B)dG(B)}{kV}$  so that Corollary 2 applies. In this case if  $L_1^* > kV$  and  $L_2^* < X_1 - L_2^*$  then by Corollary 2, we have that  $P(L, 0) = kV$  for all  $L \leq L_1^*$ , and  $P(L_2^*, 0) = X_1 - L_2^* < L_2^*$ . Since  $L_1^* > kV$  and  $P$  is increasing and continuous, this implies that (a)  $P(kV, 0) = kV$  and (b) there exists an  $L'$  with  $L_1^* < L' < L_2^*$  such that  $P(L, 0) < L$  for  $L \in (kV, L')$  and  $P(L', 0) = L'$ . By Proposition 2 this implies that monetary policy will be ineffective over the region  $R \in (\bar{B}^{-1}(kV), \bar{B}^{-1}(L'))$  and effective at both  $R = \bar{B}^{-1}(kV)$  and  $R = \bar{B}^{-1}(L')$  as stated in the proposition.

Consider then the conditions under which  $L_1^* > kV$  and  $L_2^* < X_1 - L_2^*$ . Since  $L_1^*$  satisfies  $\frac{\int_0^{L_1^*} (X_1-B)dG(B)}{kV} = \alpha$ , we have that if  $\alpha > \frac{\int_0^{kV} (X_1-B)dG(B)}{kV}$ , then  $L_1^* > kV$ . Second, since  $L_2^*$  satisfies  $\frac{\int_0^{L_2^*} (X_1-B)dG(B)}{X_1-L_2^*} = \alpha$ , we have that if  $\alpha < \frac{\int_0^{X_1/2} (X_1-B)dG(B)}{X_1/2}$  then  $L_2^* < \frac{X_1}{2}$ . This implies that  $L_2^* < X_1 - L_2^*$ . Put together, if  $\frac{\int_0^{kV} (X_1-B)dG(B)}{kV} < \alpha < \frac{\int_0^{X_1/2} (X_1-B)dG(B)}{X_1/2}$  then  $L_1^* > kV$  and  $L_2^* < X_1 - L_2^*$ . In addition, we require Corollary 2 to hold, which necessitates  $\alpha < \frac{\int_0^{X_1-kV} (X_1-B)dG(B)}{kV}$ .

Now, by L'Hôpital's rule, as  $kV$  converges to zero,  $\frac{\int_0^{kV} (X_1-B)dG(B)}{kV}$  converges to  $X_1 g(0)$  which is equal to zero due to the fact that  $g(0) = 0$ . (As can be seen, the condition that  $g(0) = 0$  implies that as  $kV$  converges to zero, aggregate insider liquidity decreases sufficiently quickly that the maximal insider demand for assets  $\frac{\int_0^{kV} (X_1-B)dG(B)}{kV}$  converges to zero as well. It is this that guarantees that for any  $\alpha$ , if  $kV$  is sufficiently small, monetary policy will be ineffective over low levels of loan supply.)

Continuing with the proof, for sufficiently low  $kV$  it is clear that  $\frac{\int_0^{X_1/2} (X_1-B)dG(B)}{X_1/2} < \frac{\int_0^{X_1-kV} (X_1-B)dG(B)}{kV}$ .

Thus, for sufficiently low  $kV$ , the set  $[\frac{\int_0^{kV} (X_1 - B) dG(B)}{kV}, \frac{\int_0^{X_1/2} (X_1 - B) dG(B)}{X_1/2}]$  is nonempty and, further, for any  $\alpha$  in it, Corollary 2 applies. Since, by construction, for any  $\alpha$  in this set we have that  $L_1^* > kV$  and  $L_2^* < X_1 - L_2^*$ , as shown above, for any such  $\alpha$  there exist  $R_1$  and  $R_2$  such that monetary policy is effective at  $R_1$  and  $R_2$  but ineffective at  $(R_1, R_2)$ .

**Proof of Proposition 7.** We begin by showing that the set of search costs  $C$  satisfying  $P(0, 0) < C \leq P(L^*, 0) - L^*$  is nonempty. To see this define the function  $H(L) = P(L, 0) - L$ . Since  $H(0) = P(0, 0)$  and  $H(L^*) = P(L^*, 0) - L^*$ , we need to show that  $H(L^*) > H(0)$ . To see this note that  $H'(L) = \frac{\partial P(L, 0)}{\partial L} - 1$ . Thus, since  $\frac{\partial P(L, 0)}{\partial L}|_{L=0} > 1$  and  $\frac{\partial P(L^*, 0)}{\partial L} = 1$ , we have that  $H'(0) > 0$  and  $H'(L^*) = 0$ . Further, because  $H''(L) = \frac{\partial^2 P(L, 0)}{\partial L^2} \leq 0$  it must be that  $H'(L) \geq 0$  for  $L \in [0, L^*]$  and  $H'(L) > 0$  over  $L \in [0, L']$  with  $L' \leq L^*$ . This implies that  $H(L^*) > H(0)$  as was desired – i.e. the set  $\{C | P(0, 0) < C \leq P(L^*, 0) - L^*\}$  is nonempty.

Consider therefore a search cost  $C$  with  $P(0, 0) < C \leq P(L^*, 0) - L^*$ . We have that  $P(0, 0) - C < 0$  and  $P(L^*, 0) - C \geq L^*$ . Since  $P(L, 0)$  is continuous and increasing in  $L$ , this implies that there exists an  $\bar{L}$  such that  $0 < \bar{L} < L^*$ , with  $P(L, 0) \leq C$  for  $L \leq \bar{L}$  and  $P(L, 0) > C$  for  $L > \bar{L}$ . This then implies that

$$\tilde{P}(L, 0) = \begin{cases} 0 & \text{if } L \leq \bar{L} \\ P(L, 0) - C & \text{if } L > \bar{L} \end{cases}$$

Since  $\bar{L} < L^*$  and  $C \leq P(L^*, 0) - L^*$  we have that  $\tilde{P}(L^*, 0) = P(L^*, 0) - C \geq L^*$ . Combining this with the fact that  $\tilde{P}(\bar{L}, 0) = 0$ ,  $\tilde{P}(L, 0)$  is increasing in  $L$ , and continuous, implies that there is an  $\tilde{L}$  between  $\bar{L}$  and  $L^*$  such that  $\tilde{P}(L, 0) < L$  for  $L < \tilde{L}$  and  $\tilde{P}(\tilde{L}, 0) = \tilde{L}$ . By Corollary 2, monetary policy is therefore ineffective at any loan supply  $R \in [0, \bar{B}^{-1}(\tilde{L})]$  and effective at  $R = \bar{B}^{-1}(\tilde{L})$ . (Note here the slight abuse of semantics in denoting monetary policy ‘ineffective’ at  $R = 0$ .)

**Proof of Proposition 8.** Based on Proposition 4(ii), the maximal loan supply which can be lent out is  $R^* = \bar{B}^{-1}(L^*)$ , where  $L^*$  is the fixed point of the pricing function  $P$  that satisfies  $P(L^*, 0; X_1) = L^*$ . Note that we write  $P$  taking as input the exogenous variable  $X_1$  which varies with changes in fiscal policy. Since  $p$  is strictly increasing in  $X_1$ , and since  $p(L; R, X_1) = P(L, 0; X_1)$ , we have that  $P$  is strictly increasing in  $X_1$  as well. Further, since under the conditions of Proposition 4(ii),  $P$  is concave with  $P(L, 0) > L$  for  $L < L^*$  while  $P(L, 0) < L$  for  $L > L^*$ , it is easy to see that  $\frac{\partial P(L^*, 0; X_1)}{\partial L} < 1$ .

To prove the proposition, differentiate both sides of the equation  $P(L^*, 0; X_1) = L^*$  by  $X_1$  and rearrange to obtain:

$$\frac{\partial L^*}{\partial X_1} = \frac{\frac{\partial P}{\partial X_1}}{1 - \frac{\partial P}{\partial L}} \quad (17)$$

Since  $\frac{\partial P}{\partial X_1} > 0$  and  $\frac{\partial P}{\partial L} < 1$  at  $L = L^*$ , we have that  $\frac{\partial L^*}{\partial X_1} > 0$ . Thus, since  $\bar{B}^{-1}$  is increasing in  $L$  and  $L$  is increasing in  $X_1$ , the maximal loan supply that can be lent out  $R^* = \bar{B}^{-1}(L^*)$  is increasing

in  $X_1$  as well.

**Proof of Proposition 9.** Denote by  $\bar{B}(\delta)$  the marginal firm obtaining financing when investing firms obtain a wealth transfer of  $\delta$  per firm at date-1. By Proposition 8,  $\bar{B}(\delta)$  is increasing in  $\delta$ . Given a transfer of  $\delta$  to each investing firm, we have that the aggregate wealth transfer to all firms is  $T(\delta) = \delta G(\bar{B}(\delta))$ . Since  $T$  is increasing in  $\delta$ , we can therefore define the function  $\delta(T) = T^{-1}(T)$ . For every wealth transfer  $T$ ,  $\delta(T)$  is the level of the individual firm transfer such that the aggregate wealth transfer is  $T$ .

Date-1 firm aggregate wealth given a wealth-transfer of  $T$  is

$$W(T) = T + \int_0^{\bar{B}(\delta(T))} (X_1 - B) dG(B) + \int_{\bar{B}(\delta(T))}^I (I - B) dG(B) \quad (18)$$

Integrating and rearranging slightly we have that

$$W(T) = T + X_1 B(\delta(T)) + I[I - \bar{B}(\delta(T))] - \int_0^I (B) dG(B). \quad (19)$$

Thus,  $\frac{dW}{dT} = 1 + (X_1 - I) \frac{d\bar{B}}{d\delta} \frac{\delta(T)}{dT} > 1$  where the inequality stems from the fact that  $X_1 > I$ , the function  $\bar{B}$  is increasing in  $\delta$ , and  $\delta$  is increasing in  $T$ .

Finally, since loan supply is greater than the maximal lending capacity of banks, for every  $\delta$ , the marginal firm obtaining financing will have a borrowing requirement of  $L^*$ , where  $L^*$  is the maximal liquidation value in the credit trap associated with a wealth transfer of  $\delta$ . This implies that  $\frac{d\bar{B}}{d\delta} = \frac{dL^*}{d\delta}$ . Similar to the proof of Proposition 8, we have that

$$\frac{dL^*}{d\delta} = \frac{\frac{\partial P}{\partial \delta}}{1 - \frac{\partial P}{\partial L}(L^*(\delta), 0; X_1 + \delta)} \quad (20)$$

Now, if  $L^*$  is a maximal liquidation value in a credit trap as in Proposition 4(ii), we have that  $\frac{\partial P}{\partial L}(L^*) < 1$ . This stems from the fact that  $P$  is concave with  $P(L^*, 0) = L^*$ ,  $P(L, 0) > L$  for  $0 < L < L^*$ , and  $P(L, 0) < L$  for  $L > L^*$ . Thus, from (20), as  $\frac{\partial P}{\partial L}(L^*(\delta), 0; X_1 + \delta)$  increases, we have that  $\frac{dL^*}{d\delta}$  increases as well. Since  $\frac{dL^*}{d\delta} = \frac{d\bar{B}}{d\delta}$  and  $\frac{dW}{dT} = 1 + (X_1 - I) \frac{d\bar{B}}{d\delta} \frac{\delta(T)}{dT}$ , we therefore have that as  $\frac{\partial P}{\partial L}(L^*(\delta), 0; X_1 + \delta)$  increases,  $\frac{dW}{dT}$  increases as well. Put differently, the strength of the fiscal multiplier is greater when the liquidity pricing effect is stronger.

**Proof of Proposition 10.** First, we show that for any pricing function  $P$  with  $P(I, 0) < I$  there exists a loan supply  $R^* \leq R_{max}$  beyond which monetary policy will be ineffective. Further,  $R^* \leq \bar{B}^{-1}(P(I, 0))$ .

To see this, note that since  $P(0, 0) \geq 0$  and  $P(I, 0) < I$ , by continuity of  $P$  there exists an  $L^*$  with  $P(L^*, 0) = L^*$ . Consider then the set of all fixed points  $\{L \mid s.t. P(L, 0) = L \text{ and } L \leq I\}$ . Since this set is bounded from above, define  $\bar{L} = \sup\{L \mid s.t. P(L, 0) = L\}$ . By continuity of  $P$ , it must be that

$P(\bar{L}, 0) = \bar{L}$ ; i.e. the supremum is a maximum as well. To see this, assume by contradiction that  $P(\bar{L}, 0) \neq \bar{L}$ . Since  $\bar{L}$  is a supremum of the fixed points of  $P(L, 0)$ , there must be a fixed point  $L_n$  in every interval  $(\bar{L} - \frac{1}{n}, \bar{L})$ . We can therefore define a sequence of fixed points  $\{L_n\}$  with  $\{L_n\} \rightarrow \bar{L}$ . Since  $P$  is continuous we have that  $\{P(L_n, 0)\} \rightarrow P(\bar{L}, 0)$ . Further, since  $\{L_n\}$  are fixed points, we have that  $\{L_n\} \rightarrow P(\bar{L}, 0)$ . However, since by construction  $\{L_n\}$  also converges to  $\bar{L}$  it must be that  $P(\bar{L}, 0) = \bar{L}$ , in contradiction to our assumption.

We therefore have that  $\bar{L} = \max\{L | s.t. P(L, 0) = L\}$ . It must be then that for any  $L > \bar{L}$ ,  $P(L, 0) < L$ . Otherwise, by continuity of  $P$  there would be an  $L' > \bar{L}$  that is a fixed point of  $P(L, 0)$  in contradiction to  $\bar{L}$  being the maximum fixed point. Since  $P(\bar{L}, 0) = \bar{L}$ , by Proposition 3 we have that  $R^* = \bar{B}^{-1}(L)$  can indeed be lent out. By the same corollary, since  $P(L, 0) < L$  for  $L > L^*$ , no  $R^* < R \leq R_{max}$  can be lent out in equilibrium, implying that monetary policy is ineffective beyond  $R^*$ . Finally, it must be that  $\bar{L} \leq P(I, 0)$ ; Otherwise, since  $\bar{L}$  is a fixed point, we have that  $P(\bar{L}, 0) = \bar{L} > P(I, 0)$ , contradicting the fact that  $P$  is increasing in  $L$  and that  $\bar{L} < I$ . Since  $\bar{L} \leq P(I, 0)$ , it must therefore be that  $R^* \leq \bar{B}^{-1}(P(I, 0))$ , as was stated above. Now define:

$$P_{max}(W) = \sup_G \{P(I, 0; G)\} \text{ s.t. } \int_0^I (I - B) dG(B) = W \quad (21)$$

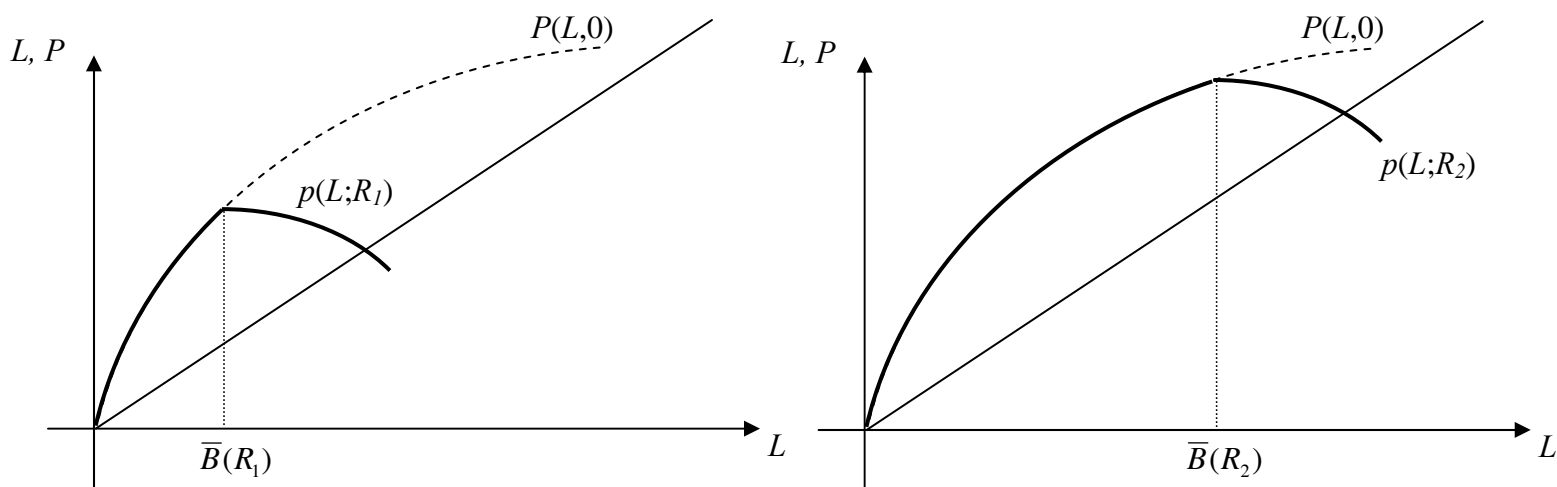
That is,  $P_{max}(W)$  is the supremum of  $P(I, 0)$  over all distributions of borrowing requirements  $G$  such that the aggregate date-0 wealth associated with  $G$  equals  $W$ .

Clearly,  $P_{max}(0)$  equals  $P_0$ , the value of collateral assuming aggregate date-0 wealth is zero and all firms obtain financing. Since by assumption  $P_0 < I$ , it must be that there exists a  $\bar{W}$  for which  $P_{max}(\bar{W}) < I$ . To see this, assume to the contrary that there is a sequence  $\{W_n\} \rightarrow 0$  and associated distributions  $\{G_n\}$  such that  $P(I, 0; G_n) \geq I$  and  $\int_0^I (I - B) dG_n(B) = W_n$  for all  $n$ . Now, since  $\int_0^I (I - B) dG(B) = I - \int_0^I B dG(B) = I - \int_0^I (1 - G(B)) dB = \int_0^I G(B) dB$  (the second equality is obtained by integration by parts), we have that  $\int_0^I G_n(B) dB = W_n$ . Define now  $G_0$  to be the distribution that places full mass on  $B = I$ : i.e. the distribution associated with zero aggregate date-0 wealth. Under the  $\mathcal{L}^1$  metric, we have that  $d(G_n, G_0) = \int_0^I (G_n(B) - G_0(B)) dB = W_n - 0 = W_n \rightarrow 0$ . Therefore,  $G_n \rightarrow G_0$  under  $\mathcal{L}^1$ . By continuity of  $P$ , we have that  $P(I, 0; G_n) \rightarrow P(I, 0; G_0) = P_0 < I$  in contradiction to  $P(I, 0; G_n) \geq I$  for all  $n$ . Thus, there must exist a  $\bar{W}$  for which  $P_{max}(\bar{W}) < I$ .

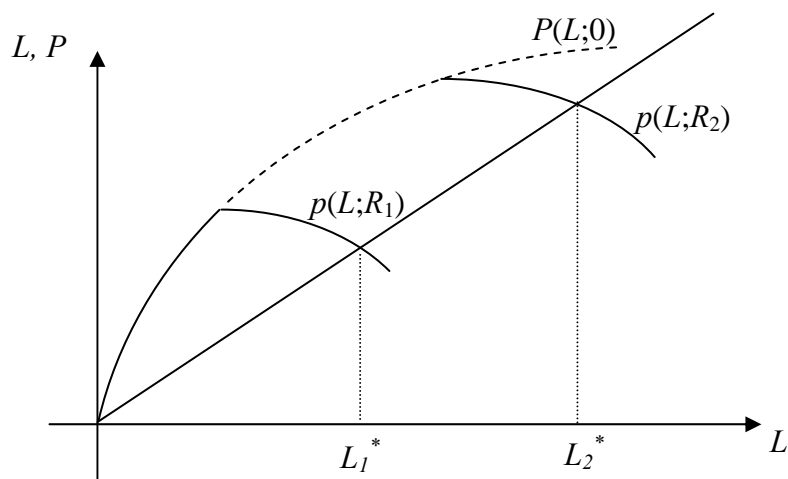
Since  $P$  is decreasing in  $W$ , we have that  $P_{max}(W) < I$  for all  $W < \bar{W}$ . By definition of the function  $P_{max}$ , we therefore have that for any distribution  $G$  with aggregate wealth  $W < W_{max}$ ,  $P(I, 0; G) \leq P_{max}(\bar{W}) < I$ . As shown above, the fact that  $P(I, 0; G) < I$ , implies that for any such  $G$  there exists an  $R^*(G)$  beyond which monetary policy is ineffective. Further, as shown above,  $R^*(G) \leq \bar{B}^{-1}(P(I, 0; G))$ . Since  $\bar{B}^{-1}$  is increasing and  $P(I, 0; G) \leq P_{max}(\bar{W})$ , we have that  $\bar{B}^{-1}(P(I, 0; G)) \leq \bar{B}^{-1}(P_{max}(\bar{W}))$ . Thus,  $R^*(G) \leq \bar{B}^{-1}(P_{max}(\bar{W}))$  for any  $G$ . This implies that monetary policy is ineffective beyond  $\bar{B}^{-1}(P_{max}(\bar{W}))$  for any distribution  $G$  with aggregate wealth  $W < W_{max}$ . Finally, since  $P_{max}(\bar{W}) < I$ , we have that  $\bar{B}^{-1}(P_{max}(\bar{W})) < \bar{B}^{-1}(I) = R_{max}$ . Thus, defining  $R^* = \bar{B}^{-1}(P_{max}(\bar{W}))$  we have that there exists an  $R^* < R_{max}$  beyond which monetary policy is ineffective for all distributions  $G$  with aggregate liquidity  $W < \bar{W}$ .

**Proof of Proposition 11.** Since by assumption under  $G_1$  the economy is in a credit trap equilibrium, we have by Proposition 4 that  $L_1^*$ , the maximal collateral value in this credit trap equilibrium, satisfies  $P(L_1^*, 0; G_1) = L_1^*$  and that  $P(L, 0; G_1) < L$  for  $L > L_1^*$ . Further, the maximal aggregate lending  $R_1^*$  is equal to  $\bar{B}^{-1}(L_1^*)$ .

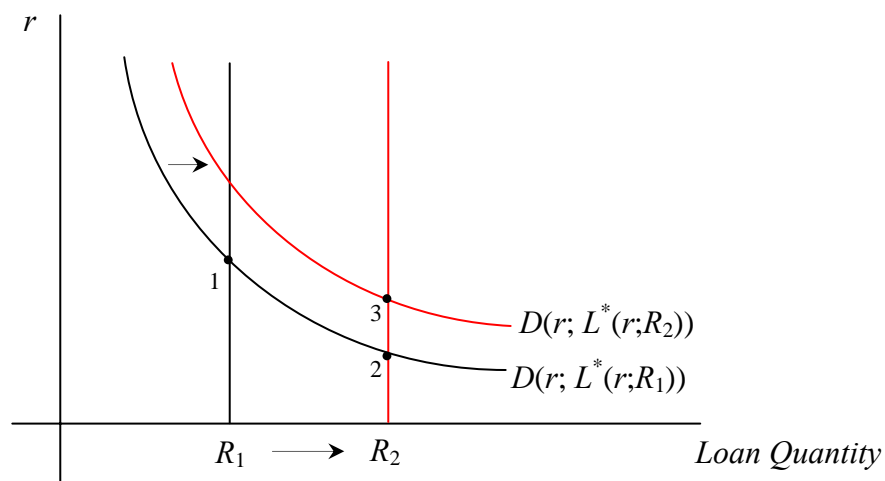
Now it is easy to see that if  $G_2$  stochastically dominates  $G_1$  (implying that date-0 liquidity in the corporate sector is higher under  $G_1$ ) equilibrium date-1 liquidity is always higher under  $G_1$  as compared to  $G_2$ . As such, we have that if  $G_2$  stochastically dominates  $G_1$ ,  $P(L, 0; G_1) > P(L, 0; G_2)$  for any  $L$ . As shown in the proof to Proposition 10, since  $P(I, 0; G_2) < P(I, 0; G_1) < I$ , under the distribution  $G_2$  there exists a maximal equilibrium collateral value  $L_2^*$  and an associated maximal equilibrium level of loan supply  $R_2^* = \bar{B}^{-1}(L_2^*)$ . Further,  $L_2^*$  is equal to the maximum of all of the fixed points  $L$  satisfying  $P(L, 0; G_2) = L$ . Since for any  $L$  we have that  $P(L, 0; G_2) < P(L, 0; G_1)$  and further for any  $L \geq L_1^*$  we have  $P(L, 0; G_1) \leq L$  it must be that  $P(L, 0; G_2) < L$  for any  $L \geq L_1^*$ . Since  $L_2^*$  exists and satisfies  $P(L_2^*, 0; G_2) = L_2^*$  (see the proof for Proposition 10) it must therefore be that  $L_2^* < L_1^*$ . Further, since the maximal loan supply that can be lent out is  $R_2^* = \bar{B}^{-1}(L_2^*)$  and  $\bar{B}^{-1}$  is increasing in  $L$ , it must be that  $R_2^* < R_1^*$ .



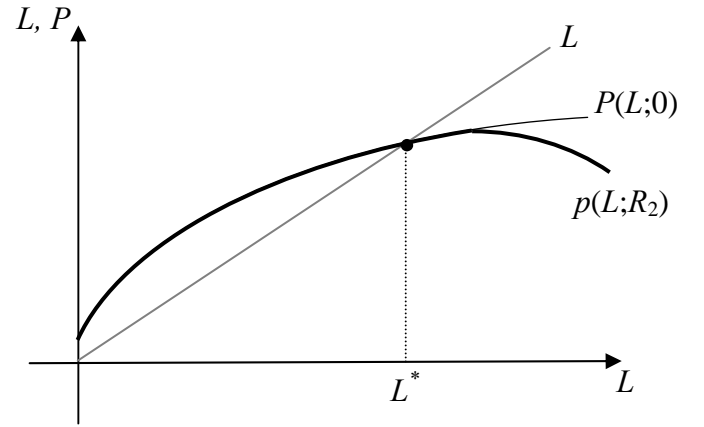
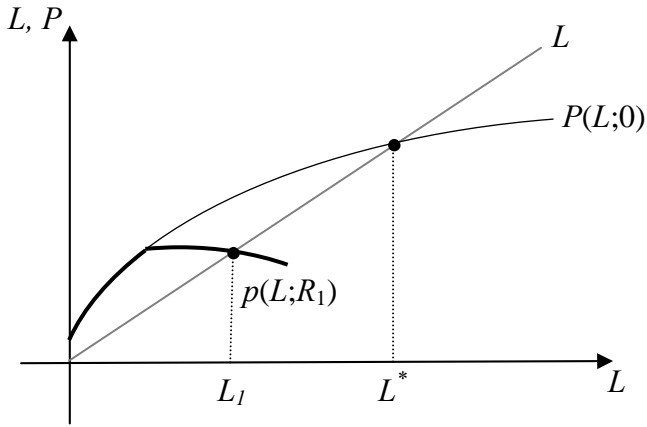
**Figure 1.** The indirect pricing function  $p(L;R)$  provides the implied price of assets assuming a liquidation value  $L$  and loan supply  $R$ . As  $R$  expands, the direct pricing function  $P(L,0)$  serves as an envelope of  $p(L;R)$ .



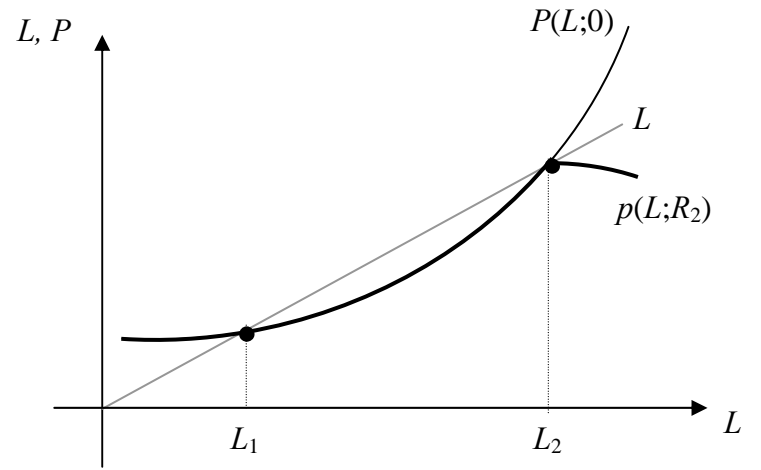
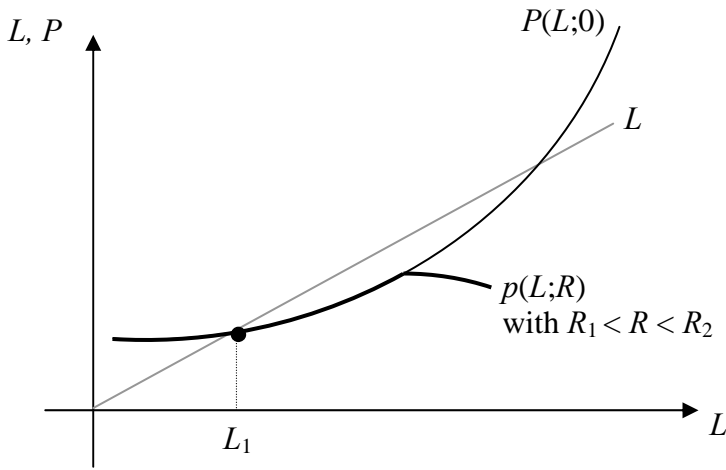
**Figure 2.** The conventional equilibrium. This figure presents the pricing function  $p(L;R)$  for two levels of loan supply,  $R_1$  and  $R_2$ . The equilibrium liquidation value increases from  $L_1^*$  to  $L_2^*$ .



**Figure 3.** The market for loanable funds: aggregate loan supply and aggregate loan demand as a function of interest rate  $r$  for two levels of loan supply,  $R_1$  and  $R_2$ . An increase in  $R$  shifts out both loan supply and effective loan demand.

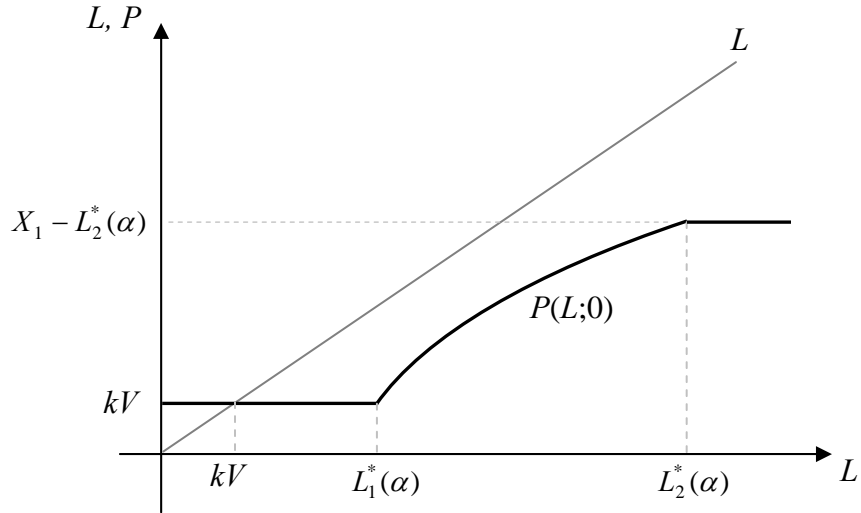


**Figure 4.** The credit trap equilibrium. The pricing function  $p(L;R)$  represents the implied price of assets assuming a liquidation value  $L$  and loan supply  $R$ . Increases in loan supply beyond  $R^* = \bar{B}^{-1}(L^*)$  do not increase the equilibrium value of collateral or equilibrium lending.

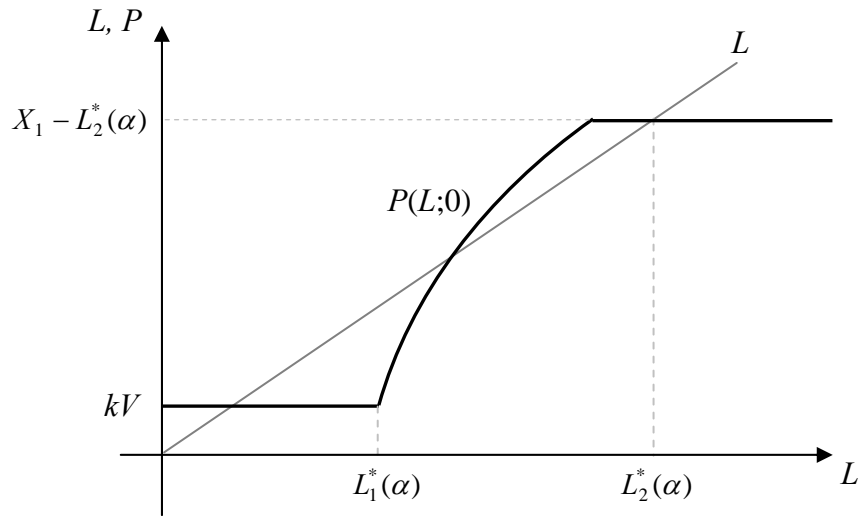


**Figure 5.** The jump-start equilibrium. The pricing function  $p(L;R)$  represents the implied price of assets assuming a liquidation value  $L$  and loan supply  $R$ . Monetary policy is ineffective over the region  $(R_1, R_2)$ , where  $\bar{B}(R_i) = L_i$ ,  $i = 1, 2$ .

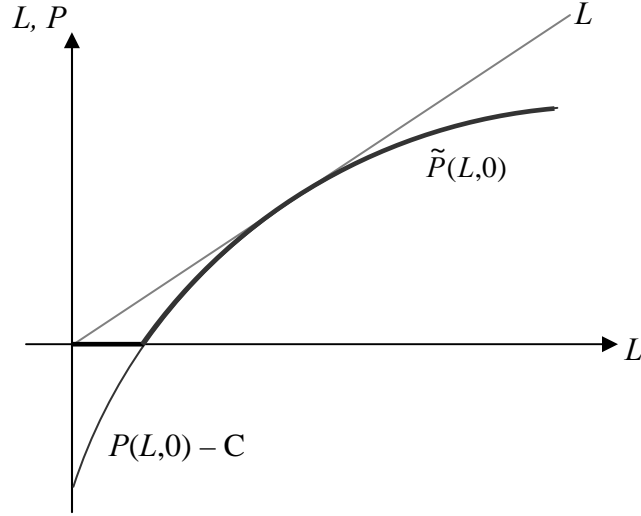




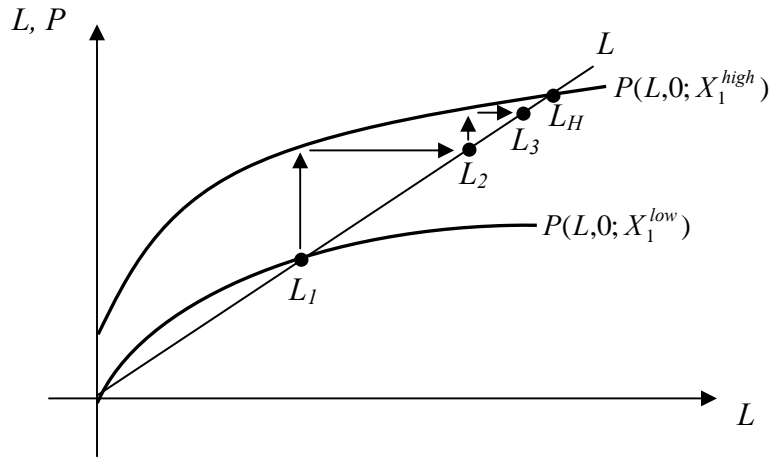
**Figure 6.** The pricing function  $P(L,0)$  in a competitive market for assets with industry-outsider valuation  $kV$  and supply of assets  $\alpha > \bar{\alpha}$ .



**Figure 7.** The pricing function  $P(L,0)$  in a competitive market for assets with industry-outsider valuation  $kV$  and supply of assets  $\alpha \in [\alpha_1, \alpha_2]$ .



**Figure 8.** Search costs and convexity of the pricing function. The pricing function  $\tilde{P}(L,0)$ , in bold, incorporates a fixed cost  $C$  of searching for industry-insider buyers. This search cost creates a convexity which brings about a jump-start, quantitative easing equilibrium.



**Figure 9.** The effect of an exogenous increase in date-1 liquidity in a credit trap equilibrium.